Chapter 5 Problems and Findings in the Parameterization Process

Since a benchmark data set should be consistent with the system of a model, incorporating a complicated structure increases requirements on data to strictly satisfy constraints and conditions included in a model. This chapter summarizes the problems we faced in the parameterization process of building the model. In Section 5.1, we will see what kind of data set is required to build the three-region, two-sector OLG/AGE model. In Section 5.2, the way how the model is calibrated to a benchmark data set is explained. Then, Section 5.3 reports the problems and difficulties we faced during the parameterization process, and Section 5.4 concludes this chapter.

5.1 Benchmark Data Set

Since it was quite ambiguous at the beginning of this research project that what kind of data set is required to build the model, model building work has started without paying too much attention to the availability of data. At this stage, we decided to continue using an artificial data set for a while to reveal fundamental characters of the model first by conducting numerous simulations. After completing that, we are planning to carry out more realistic simulations utilizing an empirical data set.

In the present situation that the basic model just started working, the necessary data can be categorized into several groups: (a) data set for a certain year on national accounts and trade flows; (b) tax/subsidy payments/revenues at the same year as the one in (a) by tax items; (c) pension related data such as contribution rates; (d) interregional portfolio flows and rates of return; (e) foreign aid flows and the composition by item; (f) data set related to demographic change such as population for a certain period, proportion of higher age marriage, time spent for child care, and time spent for schooling; (g) regional economic growth rates; (h) behavioral parameters such as elasticity of substitution and transformation, intercept and slope parameters in adjustment cost functions, shape parameters on schooling and on economic infrastructure over GDP; and data on proportion of inactive firms and Pareto shape parameter on productivity to incorporate Melitz-type heterogeneous firms.

Every kinds of data categorized in Group (a) is available in the GTAP database (Hertel 1997). Some of the ones in Group (b) might be found in the "Government Finance Statistics" prepared by IMF. Some of the ones in Group (c) might be found in the "Pension at a Glance" prepared by OECD. Considering the fact that even the FDI related data is quite limited in the present situation, Group (d) might be difficult to find since we need the data that contain information on ownership of assets at stock level. However, there might be several research projects that tried to derive the data utilizing the "International Financial Statistics" by IMF and other data sources. Foreign aid related data categorized in Group (e) might be obtained from the "International Development Statistics" prepared by OECD. The ones categorized in Group (f) might be found in several resources presented by UN. Group (g) and others may be found in several resources presented by the World Bank. Some of the ones in Group (h) can be found in the aforementioned GTAP database. Rest of Group (h) and some of Group (g) should be found in literatures conducting empirical research. Balistreli *et al.* (2011) tries to estimate parameters related to the Melitz-type formulation such as categorized in Group (g).

As mentioned, we are working with an artificial data set that includes three regions and two sectors. Regions are totally symmetric and identical at this stage. Utilizing a benchmark data set shown above, other information and parameters are all derived and calibrated based on the constraints and conditions included in the model under the assumption that the global economy is in a steady state.

Let us note about the choice of the length of a period. As variables of different time-periods are interdependent, the computation burden is much larger than that for models that calculate solutions period by period (recursively dynamic or backward-looking models). Moreover, extensions of the calculation horizon increase calculation difficulty more than proportionally, and expansions of a model with respect to the number of age groups, regions, and sectors are more difficult. Assuming short interval in one period implies an increase of age groups existing in the same period. To avoid increases in dimensionality, we decided to divide an individual's life into five phases that show essentially different characters. For instance, the first working age 20-39 is distinguished from the second working age 40-59, as the period that individuals make children. The division in sectors also shows crucial difference between the one with increasing returns and another one that provides interregional shipping services. Since we are interested in the impact of rapidly aging developing economies on the global economy, we need three regions: already aged; rapidly aging; and pre-aging regions.

5.2 Calibration of the Model

The parameterization process starts with converting information given in annual terms to be the one in periodical terms, assuming that the global economy initially is in a steady state. Then, based on the population data by age groups, survival rates Ω_{js0} and the number of children z_{j0} are derived under the assumption that population growth rate is constant. Subjective discount rate for an individual also is set as:

$$\rho_j = \left(1 - \tau_j^A\right) r_{j0}. \tag{5.1}$$

This is the first step of the procedures. The parameterization process can be divided into 10 steps.

The second step is the derivations of f_{j10}^H, Δ_j^H, and h_{js0}, utilizing the information on f_{j00}^H. Using one of the first order conditions (FOCs) for a household's optimization, we can derive f_{j10}^H with the assumption of steady state that quantity variables grow at the same constant rate, and price variables stay at constant levels. Then, we get h_{js0} using Equation (4.5). Finally, equating the labor income shown in Equation (4.2) evaluated by Ω_{js0}, f_{js0}^H, h_{js0}, and z_{j0}, with information obtained from the benchmark data, we get Δ_j^H.

Next, evaluating Equations (4.7) and (4.8) with the pre-determined parameters and initial values of variables, we can derive $\bar{a}_{js0}^F$, $\bar{a}_{js0}^P$, and Λ_{j0}. This is the third step.

The fourth step is related to the alternative specifications presented by Armington, Krugman, and Melitz. As discussed in detail by Oyamada (2013), ψ_j^K, $\psi_{jj'}^M$, M_{j0}, and $E_{ijj'0}$ are calibrated simultaneously by solving the system of Equations (4.17), (4.19), (4.20), (4.22), in addition to

$$TF_{ijj'0} = \left(1 - \xi_{jj'0}\right) M_{j'0} p_{ijj'0} E_{ijj'0}, \tag{5.2}$$

after $\nabla^{M}_{jj'0}$ and $p_{ijj'0}$ are derived using Equations (4.21) and (4.20), respectively. Then, p^{M}_{ij0} and $\alpha^{T}_{ijj'}$ can be obtained by:

$$p^{M}_{ij0} = \frac{\sum_{j'}\left(1+\tau^{M}_{ijj'}\right)\left(1+\tau^{T}_{ijj'}\right)\left(1-\xi_{jj'0}\right)M_{j'0}p_{ijj'0}E_{ijj'0}}{\sum_{j'}\left(1-\xi_{jj'0}\right)M_{j'0}E_{ijj'0}}, \tag{5.3}$$

and Equation (4.17). Note that the value of p^{M}_{ij0} differs among three specifications since $\xi_{jj'0} = 0$ in the Krugman and Armington types, and $M_{j'0} = 1$ in the Armington type.

In the fifth step, quantities of regional composites of intermediate input $O_{ii'j0}$, private consumption C^{P}_{ij0}, government consumption C^{G}_{ij0}, private GFCF F^{P}_{ij0}, GFCF for economic infrastructure F^{E}_{ij0}, and GFCF for social infrastructure F^{S}_{ij0} are obtained based on the p^{M}_{ij0} obtained in the previous step, which is different among three specifications. It implies that prices of sectoral composite p^{O}_{ij0}, p^{C}_{j0}, p^{G}_{j0}, p^{P}_{j0}, p^{E}_{j0}, and p^{S}_{j0} also have different values.

The sixth step calibrates b_{j0}, $\bar{a}_{js0}$, β^{H}_{j}, β^{Z}_{j}, and β^{B}_{j}. Using Equations (4.2) and (4.4) along with the information on $\hat{c}^{P}_{j10}$, which is obtained in the previous step, we obtain b_{j0} and $\bar{a}_{js0}$. Then, β^{H}_{j}, β^{Z}_{j}, and β^{B}_{j} can be calibrated using FOCs for a household's optimization with the assumption of steady state.

Once $\bar{a}_{js0}$ is calibrated, we may derive $A^{K}_{ijj'0}$ utilizing information on interregional portfolio flows and government savings, and

$$A^{T}_{j'0} = \sum_{s=1}^{S}\left(N_{j's0}\left[a_{j's0} + a^{F}_{j's0} + \left(\frac{1}{1+\gamma}\right)b_{j'0} + \left\{1 + \left(1-\tau^{A}_{j}\right)r_{j'0}\right\}\left(\frac{1}{1+\gamma}\right)^{2} b_{j'0}\right]\right). \tag{5.4}$$

Then, either τ^{V}_{ij} or τ^{F}_{ij} should be calibrated using FOCs for an enterprise's optimization and Equation (4.37). This is the seventh step.

In the eighth step, almost all the parameters and initial values related to financial portfolio can be calibrated utilizing the information on $A^{K}_{ijj'0}$ and $A^{G}_{jj'0}$, and Equations (4.31) through (4.34) with FOCs for investment trust bank's optimization problems. In addition, parameters related to public expenditure such as Ξ^{A}_{j}, Ξ^{C}_{j}, Ξ^{F}_{j}, Ξ^{P}_{j}, and Ξ^{G}_{j}, can also be obtained.

The ninth step calibrates $\overline{K}^P_{ij0}$, p^K_{ij0}, and δ^P_{ij} using Equation (4.11) and FOCs for and enterprise's optimization along with the information on τ^V_{ij} or τ^F_{ij}, and $\hat{F}^P_{ij0}$.

In the final step, all of the share parameters and unit coefficients for production functions and composite commodity aggregators are calibrated following the usual procedures in AGE modeling, based on the values of prices that differs among three trade related specifications.

5.3 Problems and Findings

In the parameterization process, explained in the previous section, we faced several problems or difficulties, and found some interesting characters of the model. This section summarizes those findings.

Let us start with Equation (5.1). It shows a change in the asset income tax rate τ^A_j have impact similar to a change in the overall rate of return. This implies that τ^A_j can be used as an item that cancels out the effect of changes in the rate of return especially in the case when an economy suffers from negative impact.

Until the sixth step shown in the previous section, any serious problem may not arise. However, in the calibration process of bequest b_{j0} and initial asset holdings by age groups $\bar{a}_{js0}$, we may see negative values of those variables. It comes from the comparative volumes of private consumption and GFCG given by the data, or other set of information on rates of return and tax rates. Since an individual is designed to make bequest account at the end of his/her third age period $s = 2$, the volume of bequest sometimes exceeds the level of savings. This tendency also is promoted by the existence of pensions. At the end of the third age period, people is going to borrow to finance the bequest expecting that he/she will receive two kinds of pensions in the next age period. This happens in the case that the volume of bequest, required by the constraints of the model, is relatively large. When the volume of bequest is small, savings in the younger age period tend to be not enough to cover consumption.

The next problem arises in the process of calibrating corporate tax rate τ^V_{ij} or the rate of investment tax credit τ^F_{ij}. The values of these rates often become negative or exceed unity. This also might be the result of the model's complexity.

In the ninth step, one may see the physical depreciation rate δ^P_{ij} that takes a negative value or exceeds unity. This comes from the balance between private GFCF and rate of return. In addition, if the calibrated value of δ_{ij}^{P} is small, it is not so favorable because it is unrealistic with the model that has 20-year time intervals. 3.5% of annual depreciation becomes 99% for the case of 20-year period.

When these kinds of problem arise, it requires the benchmark data set to be adjusted. Our experience tells that the adjustment tend to be drastic. Especially, relative balance in the volume between private consumption, private GFCF, and labor income (or operating surplus) is important. The point is that an empirical data set often fails to satisfy the conditions a model requires. Once the data is adjusted, the merit of using empirical data might be lost. This is one of the reasons we decided to work with an artificial data set for a while.

Another problem is in the calibration procedure assuming that the global economy is in a steady state. It is quite unrealistic to assume steady state to the global economy, because almost all the data, as well as the information on demographic structures, show the global economy is in an adjustment process. Since the model developed in this research project is handling demographic structures, we have to find a way to calibrate the model to a non-stationary growth path, while it must be a quite tough challenge especially for a model that has a number of regions.

5.4 Concluding Remarks

In this section, we outlined what kind of data set is necessary in building the model, as well as the way how the model is parameterized. In the process of model building, we experienced an overwhelming amount of difficulties and problems. Some of the parameters that need to stay in between zero and one are often calibrated negative or exceeding one. Most of the problems arise from the complexity of the model. Increase in the constraints also increases the conditions a benchmark data set must satisfy. In the present situation, data adjustment seems to be the only way to solve this kind of problem. Another problem is the assumption of steady state that is utilized when calibrating the model. We need an effort to find a way to calibrate the model to a non-stationary growth path.

Those problems are still there and have not yet been overcome. One of the purposes of this chapter is to share our experience with model builders in the AGE community, and start exchanging ideas to tackle the difficulties. We hope this note will motivate experienced model builders to start discussions to find a solution.