Part II The Model

Chapter 4 Basic Structure and Major Assumptions

In this chapter, we outline the basic structure and the major assumptions of the three-region, two-sector OLG/AGE model developed for this research project. This chapter is organized as follows. In Section 4.1, we present an overview of the economic environment. Sections 4.2 through 4.6 explain the structures and assumptions of sub-models by economic agent, such as households, enterprises, governments, and investment trust banks. Then, Section 4.7 presents equilibrium conditions to close the model. Finally, Section 4.8 concludes this chapter.

4.1 Environment

Let us consider three open economies linked together that respectively produce two types of commodity indexed $i = 1, 2$. Sector $i = 1$ is the manufacturing sector that exhibits increasing returns to scale (IRTS), while Sector $i = 2$ is the non-manufacturing sector that has constant returns. Sector $i = 2$ supplies a portion of its output as interregional shipping service. Regions are indexed $j = 1, 2, 3$.

Each time period $t = 0, 1, \ldots, T$ includes 20 years. The terminal period T is set to 49. In a region, an individual may live five periods so that five generations (age groups) indexed $s = 0,1, \ldots, S$ exist in the same time period. Hence, the terminal age period S is set to 4. For each age group, survival rate Ω_{jst} is considered to define life expectancy. Changes in Ω_{jst} incur demographic changes through two channels. One is the channel that directly affects demographic structure changing population of an age group. Another one indirectly affects through the personal life plan of an individual. When the life expectancy becomes longer, one may increase savings to prepare for his/her old age. Then, he/she is going to increase working time while suppressing the time for child care. This kind of behavioral change affects the number of children an individual may have.

The first age period $s = 0$ corresponds to childhood when an individual chooses time allocation between schooling and leisure. While leisure time contributes to obtain higher welfare levels within the age, short schooling time brings lower productivity in the next age period that affects wage income. In the second age period $s = 1$, an individual chooses time allocation between working, child care, and schooling to accumulate personal human capital. The sources of welfare in this age period are consumption and having children that determines the time spent for child care. While most of the decision-makings are similar to the second age period, people in the third age period $s = 2$, stops schooling.

In the second and third age periods, each individual contributes a fraction of his/her income to PAYG type pension system, in addition to the FF type pension reserve. Another important task in the third age period is to make the bequest account for descendants. From the end of the age period $s = 2$, bequest is deposited to the account until the holder dies. By the death of an individual, bequest is transferred to the next generations. On the other hand, regular assets and FF pension reserves will be shared by the people in the same age group when an individual dies.

By the end of the third age period $s = 2$, people retire from working. Then, in the fourth age period $s = 3$, an individual receives the FF pension reserves all at once. One also receives PAYG type pension in his/her fourth and terminal age periods $s = 3, S$, based on his/her contribution record in the working age.

In the working and retired age periods, people determine the level of consumption and savings. Consumption is the main source of welfare after people start working. A child in the first age period $s = 0$ does not consume since feeding and providing things to one's children is also a task of parents. An individual in the third age periods also obtain welfare from leaving bequest.

In every region, the services of effective labor and private capital stock are employed in the production of two types of commodity. The private capital is accumulated by putty-clay type technology, while the effective labor is mobile across sectors. The productivity is enhanced by two types of public capital, economic and social infrastructure. Economic infrastructure that can be regarded as roads, bridges, ports, and so on, directly promotes Hicks neutral type technical change. On the other hand, social infrastructure that can be regarded as schools, hospitals, training facilities, and so on, promotes the accumulation efficiency of personal human capital that affects supply of effective labor. In consequence, per capita growth rate in every region is determined endogenously.

The two kinds of commodity produced in every region are sold in both intra- and interregional markets. Commodities are not treated as homogeneous across regions but as imperfect substitute for that of another to handle cross-hauling, which is often observed in real data, between economies that have similar technologies and factor endowments. To incorporate intra-industry trade, the so-called "Armington assumption" has been widely adopted by conventional AGE models of global trade. Since many theoretical illustrations of product differentiation have been proposed in the steady advance of new trade theory, we enable the model to flexibly choose three kinds of trade specifications presented by Armington (1969), Krugman (1980), and Melitz (2003), in Sector $i = 1$. The latter two assume existence of monopolistic competition among firms to describe cost reductions brought by economies of scale and increased variety obtained through additional imports. Further, Melitz type specification additionally incorporates endogenous productivity growth among heterogeneous firms.

The government in every region accumulates aforementioned two kinds of public capital by public investment, provides foreign aid and compensation for PAYG fund, and consumes based on the revenue from taxes, receipt of foreign aid, and negative government savings. The negative government saving is financed by issues of government securities, which accumulate to sovereign debt.

Finally, regular assets, FF pension reserves, and funds deposited to bequest accounts held by individuals are all collected by regional investment trust bank, and invested to every local asset markets beyond regional boundary. Departing from the conventional growth models, which often assume perfect interregional capital market, corporate capital and government securities issued in every region are assumed to be imperfect substitutes, similar to the traded commodities. Therefore, those financial instruments have their own rates of return that are evaluated with risk premiums by asset holders.

The reasons why we presume imperfectly substituting financial instruments are: (a) to handle home bias that is often observed in real data; (b) developing economies do not have such perfectly efficient capital market; and (c) it becomes difficult to capture the problem we are interested in if the perfectly mobile capital is assumed. Since we are going to focus on shortages of capital compared to labor in young region and glut of savings in aged region, modeling frictions in interregional capital movement is absolutely essential. The problem we are questioning is automatically solved in a model with perfect capital mobility.

4.2 Households and Pension System

Given the rate of return on composite asset r_{jt}, rental price of effective labor w_{jt}^{L}, and composite price of consumption good p_{jt}^{C}, an individual in each region chooses time paths of consumption $\hat{c}_{jst}^{P}$ and savings a_{jst}, levels of bequest b_{jt} and schooling time f_{jst}^{H} in childhood $s = 0$ and the first working age $s = 1$, and number of children z_{st} to have that maximizes his/her felicity u_{jt} defined as the sum of discounted temporal utility. The temporal utility is discounted by the individual's positive and constant rate of time preference ρ_j, which is identical to all individuals in a region.

The utility function for an individual who is born in time period t is assumed to be homogenous and additively separable with constant elasticity of marginal utility:

$$u_{jt} = \beta_j^H \ln\left(1 - f_{j0t}^H\right) + \sum_{s=1}^{S} \left(\frac{1}{1+\rho_j}\right)^s \ln \hat{c}_{jst+s}^P + \beta_j^Z \left(\frac{1}{1+\rho_j}\right) z_{jt+1} + \beta_j^B \left(\frac{1}{1+\rho_j}\right)^2 b_{jt+2}, \tag{4.1}$$

where β_j^H, β_j^Z, and β_j^B are weights for utility. Since children are assumed to be made in the first working age period $s = 1$, i.e., age 20 to 39, and bequest is prepared in the end of the second working age period $s = 2$, the first term in the right-hand-side corresponds to $s = 0$, the second to $1 \leq s \leq S$, the third to $s = 1$, and the fourth to $s = 2$, respectively.

Let us see the each individual's flow budget constraint. In an OLG model, it is necessary to consider two types of terminal period. One is the terminal age period $s = S$, and another is the terminal time period $t = T$ of analysis. Therefore, we need to set up three types of budget constraint: (a) the constraint for an individual who does not live beyond the terminal time period $t = T$; (b) the constraint for an individual who live beyond the terminal time period $t = T$, and (c) the constraint for an individual's terminal age period $s = S$. These three are as follows:

$$a_{jst} = \bar{a}_{js0} \qquad (\bar{a}_{js0}\text{: given}) \qquad (t = 0)$$

$$+0 \qquad (s=0)$$

$$+\left(1-\tau_j^B\right)\upsilon_j \frac{N_{js+1t-1}}{N_{jst}}\left(1-\frac{\Omega_{js+2t}}{\Omega_{js+1t-1}}\right) b_{jt-1} \qquad (s=1)$$

$$+\left(\begin{array}{c} \left(1-\tau_j^B\right)\left[\begin{array}{c} \upsilon_j \frac{N_{js+1t-1}}{N_{jst}}\left(1-\frac{\Omega_{js+2t}}{\Omega_{js+1t-1}}\right)\left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}b_{jt-2} \\ +\left(1-\upsilon_j\right)\frac{N_{jst-1}}{N_{jst}}\left(1-\frac{\Omega_{js+1t}}{\Omega_{jst-1}}\right) b_{jt-1} \end{array}\right] \\ +\frac{\Omega_{js-1t-1}}{\Omega_{jst}}\left[\begin{array}{c} \left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}a_{js-1t-1} \\ +\left(1-\tau_j^L\right)\left(1-\phi_j^F-\phi_j^P\right)w_{jt-1}^L h_{js-1t-1} \\ \times\left\{1-\chi_j\left(1-\upsilon_j\right)z_{jt-1}-f_{js-1t-1}^H\right\} \\ +\tau_j^Z w_{jt-1}^L h_{js-1t-1}\chi_j\left(1-\upsilon_j\right)z_{jt-1} \\ -p_{jt-1}^C \hat{c}_{js-1t-1}^P \end{array}\right] \end{array}\right) \qquad (s=2)$$

$$+\left(\begin{array}{c} \left(1-\tau_j^B\right)\left[\begin{array}{c} \upsilon_j \frac{N_{js+1t-1}}{N_{jst}}\left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}\left\{1+\left(1-\tau_j^A\right)r_{jt-2}\right\}b_{jt-3} \\ +\left(1-\upsilon_j\right)\frac{N_{jst-1}}{N_{jst}}\left(1-\frac{\Omega_{js+1t}}{\Omega_{jst-1}}\right)\left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}b_{jt-2} \end{array}\right] \\ +\frac{\Omega_{js-1t-1}}{\Omega_{jst}}\left[\begin{array}{c} \left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}a_{js-1t-1} \\ +\left(1-\tau_j^L\right)\left(1-\phi_j^F-\phi_j^P\right)w_{jt-1}^L h_{js-1t-1}\left(1-\chi_j\upsilon_j z_{jt-2}\right) \\ +\tau_j^Z w_{jt-1}^L h_{js-1t-1}\chi_j\upsilon_j z_{jt-2} \\ -p_{jt-1}^C \hat{c}_{js-1t-1}^P \\ -b_{jt-1} \end{array}\right] \end{array}\right)$$

$$(s=3)$$

$$+\left(\begin{array}{c} \left(1-\tau_j^B\right)\left(1-\upsilon_j\right)\frac{N_{jst-1}}{N_{jst}}\left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}\left\{1+\left(1-\tau_j^A\right)r_{jt-2}\right\}b_{jt-3} \\ +\frac{\Omega_{js-1t-1}}{\Omega_{jst}}\left[\begin{array}{c} \left\{1+\left(1-\tau_j^A\right)r_{jt-1}\right\}\left\{a_{js-1t-1}+\left(1-\tau_j^I\right)a_{js-1t-1}^F\right\} \\ +\left(1-\tau_j^I\right)\Lambda_{jt-1}a_{js-1t-1}^P \\ -p_{jt-1}^C \hat{c}_{js-1t-1}^P \end{array}\right] \end{array}\right)$$

$$(s=S);$$

$$(4.2)$$

$$\left(1+\gamma_j\right)a_{jsT}$$

$$= 0 \qquad (t=T,\ s=0)$$

$$+\left(1-\tau_j^B\right)\left(\frac{1}{1+\gamma_j^N}\right)\upsilon_j \frac{N_{js+1T}}{N_{jsT}}\left(1-\frac{\Omega_{js+2T}}{\Omega_{js+1T}}\right) b_{jT} \qquad (t=T,\ s=1)$$

$$
+\left(\begin{array}{c}
(1-\tau_j^B)\left(\frac{1}{1+\gamma_j^N}\right)\left[\begin{array}{c}\left\{\frac{1+(1-\tau_j^A)r_{jT}}{1+\gamma_j}\right\}\upsilon_j\frac{N_{js+1T}}{N_{jT}}\left(1-\frac{\Omega_{js+2T}}{\Omega_{js+1T}}\right)\\ +(1-\upsilon_j)\left(1-\frac{\Omega_{js+1T}}{\Omega_{jsT}}\right)\end{array}\right]b_{jT}\\
+\frac{\Omega_{js-1T}}{\Omega_{jsT}}\left[\begin{array}{c}\{1+(1-\tau_j^A)r_{jT}\}a_{js-1T}\\ +(1-\tau_j^L)(1-\phi_j^F-\phi_j^P)w_{jT}^L h_{js-1T}\\ \times\{1-\chi_j(1-\upsilon_j)z_{jT}-f_{js-1T}^H\}\\ +\tau_j^Z w_{jT}^L h_{js-1T}\chi_j(1-\upsilon_j)z_{jT}\\ -p_{jT}^C\hat{c}_{js-1T}^P\end{array}\right]
\end{array}\right)
$$

$$(t=T,\ s=2)$$

$$
+\left(\begin{array}{c}
(1-\tau_j^B)\left\{\frac{1+(1-\tau_j^A)r_{jT}}{1+\gamma_j}\right\}\left(\frac{1}{1+\gamma_j^N}\right)\left[\begin{array}{c}\left\{\frac{1+(1-\tau_j^A)r_{jT}}{1+\gamma_j}\right\}\upsilon_j\frac{N_{js+1T}}{N_{jsT}}\\ +(1-\upsilon_j)\left(1-\frac{\Omega_{js+1T}}{\Omega_{jsT}}\right)\end{array}\right]b_{jT}\\
+\frac{\Omega_{js-1T}}{\Omega_{jsT}}\left[\begin{array}{c}\{1+(1-\tau_j^A)r_{jT}\}a_{js-1T}\\ +(1-\tau_j^L)(1-\phi_j^F-\phi_j^P)w_{jT}^L h_{js-1T}(1-\chi_j\upsilon_j z_{jT})\\ +\tau_j^Z w_{jT}^L h_{js-1T}\chi_j\upsilon_j z_{jT}\\ -p_{jT}^C\hat{c}_{js-1T}^P\\ -b_{jT}\end{array}\right]
\end{array}\right)
$$

$$(t=T,\ s=3)$$

$$
+\left(\begin{array}{c}
(1-\tau_j^B)\left\{\frac{1+(1-\tau_j^A)r_{jT}}{1+\gamma_j}\right\}^2\left(\frac{1}{1+\gamma_j^N}\right)(1-\upsilon_j)b_{jT}\\
+\frac{\Omega_{js-1T}}{\Omega_{jsT}}\left[\begin{array}{c}\{1+(1-\tau_j^A)r_{jT}\}\{a_{js-1T}+(1-\tau_j^I)a_{js-1T}^F\}\\ +(1-\tau_j^I)\Lambda_{jT}a_{js-1T}^P\\ -p_{jT}^C\hat{c}_{js-1T}^P\end{array}\right]
\end{array}\right)
$$

$$(t=T,\ s=S);$$

(4.3)

and

$$p_{jt}^C\hat{c}_{jSt}^P=\{1+(1-\tau_j^A)r_{jt}\}a_{jSt}+(1-\tau_j^I)\Lambda_{jt}a_{jSt}^P \qquad (s=S), \qquad (4.4)$$

where a_{jst} is composite asset held by an individual,

$\bar{a}_{js0}$ is composite asset held by an individual at the initial time period,

a^F_{jst} is FF pension reserve by an individual,

a^P_{jst} is contribution record of PAYG pension by an individual,

h_{jst} is personal stock of human capital,

γ^N_j is post-terminal population growth rate,

N_{jst} is population by age group,

Ω_{jst} is survival rate,

χ_j is time for child care,

υ_j is proportion of higher age marriage,

ϕ^F_j is contribution rate for FF pension,

ϕ^P_j is contribution rate for PAYG pension,

τ^B_j is inheritance tax rate,

τ^L_j is labor income tax rate,

τ^I_j is pension income tax rate,

τ^Z_j is child care tax credit (subsidy), and

Λ_{jt} is the level of PAYG pension benefits.

In addition, personal stock of human capital accumulation follows the rule below:

$$h_{jst} = \begin{cases} 0 & (s = 0,3,S) \\ +\Delta^H_j \left(f^H_{js-1t-1}\right)^{\omega^H} \left(\frac{K^S_{jt-1}}{\sum_{s'} N_{js't-1}}\right) + \left(1-\delta^H_j\right) h_{js-1t-1} & (s = 1,2), \end{cases} \quad (4.5)$$

where K^S_{jt} is stock of social infrastructure,

δ^H_j is depreciation rate of personal human capital,

ω^H is shape parameter on schooling, and

Δ^H_j is unit coefficient.

Notice that the stock of social infrastructure K^S_{jt} is divided by the total population $\sum_{s'} N_{js't}$. This implies that we assume that the schools, hospitals, training facilities, and so on, will be congested and their availability declines as population grows.

Note that the levels of asset holdings a_{jst}, FF pension reserve a^F_{jst}, and contribution record of PAYG a^P_{jst} are measured at the beginning of a time period, while the payments for labor service supplied and composite commodity consumed in a time period are made at the end of the period. Bequest b_{jt} is transferred to the next generations at the beginning of a time period by the death of an individual, while pensions are provided at the end of the period to support payments.

At the beginning of age period $s = 1$, an individual receives bequest from his/her dying young parent. Precisely, the transfer is made to an individual's asset account of the local investment trust bank from the parent's bequest account. Manipulating the fund as assets, an individual makes children, goes to school, supplies effective labor, and consumes. The value of asset holdings plus the balance between income and payment, measured at the end of the period $s = 1$, becomes the asset holdings of the next age period. Then, at the beginning of age period $s = 2$, an individual receives bequest again from his/her parent.

In the age period $s = 2$, an individual creates bequest account for descendants instead of making children. At the end of the period, he/she retires from working. At the beginning of retired age period $s = 3$, an individual receives bequest again from his/her parent. In the period, an individual receives both FF and PAYG pensions instead of working. As mentioned in the previous section, FF pension is disbursed all at once, while PAYG pension can be received as long as one is alive. At the end of terminal age period $s = S$, an individual terminate his/her asset account, clears the balance of payment and income, and dies. The reason why the survival rates appear in some part of the budget constraints is because the assets held by dying young are shared by other individuals in the same age group.

As mentioned, an individual in each region chooses time path of consumption $\hat{c}^P_{jst}$ and savings a_{jst}, levels of bequest b_{jt} and schooling time f^H_{jst} in childhood $s = 0$ and the first working age $s = 1$, and number of children z_{st} to have to maximize the objective function shown as Equation (4.1) subject to Equations (4.2), (4.3) and (4.4). The accumulation of personal human capital expressed as Equation (4.5), transition of population:

$$N_{jst+s} = \Omega_{jst+s}\left\{N_{j1t-1}\upsilon_j z_{1t-1} + N_{j1t}\left(1 - \upsilon_j\right)z_{1t}\right\}, \tag{4.6}$$

as well as FF pension reserve:

$$\begin{aligned} a^F_{jst} = \ & \bar{a}^F_{js0} \qquad (\bar{a}^F_{js0}\text{: given}) && (t = 0) \\ & +0 && (s = 0,1,S) \end{aligned}$$

$$+\frac{\Omega_{js-1t-1}}{\Omega_{jst}}\phi_j^F w_{jt-1}^L h_{js-1t-1}\{1-\chi_j(1-\upsilon_j)z_{jt-1}-f_{js-1t-1}^H\}\ (s=2)$$

$$+\frac{\Omega_{js-1t-1}}{\Omega_{jst}}\begin{bmatrix}\{1+(1-\tau_j^A)r_{jt-1}\}a_{js-1t-1}^F\\ +\phi_j^F w_{jt-1}^L h_{js-1t-1}(1-\chi_j\upsilon_j z_{jt-2})\end{bmatrix}\qquad (s=3),\ (4.7)$$

contribution record of PAYG pension:

$$\begin{aligned}a_{jst}^P = &\ \bar{a}_{js0}^P \qquad (\bar{a}_{js0}^P\text{: given}) && (t=0)\\ &+0 && (s=0,1)\\ &+\phi_j^P w_{jt-1}^L h_{js-1t-1}\{1-\chi_j(1-\upsilon_j)z_{jt-1}-f_{js-1t-1}^H\} && (s=2)\\ &+[\{1+(1-\tau_j^A)r_{jt-1}\}a_{js-1t-1}^P+\phi_j^P w_{jt-1}^L h_{js-1t-1}(1-\chi_j\upsilon_j z_{jt-2})] && (s=3)\\ &+\{1+(1-\tau_j^A)r_{jt-1}\}a_{js-1t-1}^P && (s=S),\end{aligned}$$

(4.8)

and the level of PAYG pension benefits:

$$\Lambda_{jt}\equiv\frac{\sum_{s=1}^{2}N_{jst}\phi_j^P w_{jt-1}^L h_{js-1t-1}\left[1-\{\chi_j(1-\upsilon_j)z_{jt}+f_{jst}^H\}-\chi_j\upsilon_j z_{jt-1}\right]+\Xi_j^P\Theta_{jt}}{\sum_{s=3}^{S}N_{jst}a_{jst}^P},$$

where

Ξ_j^P is share of compensation for PAYG fund in fiscal budget, and
Θ_{jt} is fiscal budget,

are all determined outside the individual’s felicity maximization.

4.3 Enterprises

There is one enterprise in each sector for every region, which produces one kind of commodity. An enterprise is organized by three kinds of firms respectively engage in investment, production, and sales businesses. The investment segment makes dynamic investment plan to maximize the value of the enterprise, while the production segment determines the volumes of production, i.e., output and factor inputs to maximize temporal profit. These two segments cooperate together in solving their optimization problems. The production segment wholesales its product to the sales segment that consists of a number of dealers/merchants who may put forth their market power by marking up the sales price of the commodity in the monopolistically competitive environment when it is Sector $i = 1$. If it is perfectly competitive Sector $i = 2$, the sales business is carried out by a representative agent.

Given the rate of return on corporate capital r^K_{ijt}, rental price of effective labor w^L_{jt}, wholesale price of the product p^W_{ijt}, composite price of intermediate input p^O_{ijt}, and composite price of capital good p^P_{jt}, the investment and production segments chooses time paths of investment $\hat{F}^P_{ijt}$, gross output Q_{ijt}, intermediate input $\hat{O}_{ijt}$, and input of effective labor L_{ijt} that maximizes the value of the enterprise VE_{ij}. It is:

$$VE_{ij} = \sum_{t=0}^{T}\left(\left(\prod_{t'=0}^{t}\frac{1}{1+r^K_{ijt'}}\right)\left[\begin{matrix}\left(1-\tau^V_{ij}\right)\left\{\left(\frac{1}{1+\tau^Q_{ij}}\right)p^W_{ijt}Q_{ijt} - p^O_{ijt}\hat{O}_{ijt} - w^L_{jt}L_{ijt}\right\}\\ -\left(1-\tau^F_{ij}\right)p^P_{jt}\hat{F}^P_{ijt}\end{matrix}\right]\right)$$

$$+\left(\prod_{t'=0}^{T}\frac{1}{1+r^K_{ijt'}}\right)\left(1+\hat{\gamma}_j\right)p^{KT}_{ij}K^P_{ijT}, \tag{4.9}$$

where K^P_{ijt} is stock of corporate capital,
τ^V_{ij} is corporate tax rate,
τ^Q_{ij} is sales tax rate on wholesale,
τ^F_{ij} is investment tax credit (subsidy),
p^{KT}_{ij} is post-terminal price of corporate capital, and
$\hat{\gamma}_j$ is post-terminal overall growth rate such that $\hat{\gamma}_j = \left(1+\gamma^N_j\right)\left(1+\gamma_j\right) - 1$.

The second term of the right-hand-side corresponds to the post-terminal value of the enterprise.

In the accumulation of corporate capital, putty-clay type capital installation is assumed. We also presume the existence of Uzawa-Penrose type adjustment cost. Then, the transition of the private capital stock can be expressed as follows:

$$K^P_{ijt} = \bar{K}^P_{ij0} \qquad (\bar{K}^P_{ij0}\text{: given}) \qquad\qquad (t=0)$$

$$+\frac{1}{\eta^P_{ij}}\left[\left\{\left(\mu^P_{ij}\right)^2 + 2\eta^P_{ij}\left(\frac{\hat{F}^P_{ijt-1}}{K^P_{ijt-1}}\right)\right\}^{\frac{1}{2}} + \eta^P_{ij}\left(1-\delta^P_{ij}\right) - \mu^P_{ij}\right]K^P_{ijt-1} \quad (t \neq 0), \tag{4.10}$$

and

$$\hat{\gamma}_j + \delta_{ij}^P = \frac{1}{\eta_{ij}^P}\left[\left\{\left(\mu_{ij}^P\right)^2 + 2\eta_{ij}^P\left(\frac{\hat{F}_{ijt-1}^P}{K_{ijt-1}^P}\right)\right\}^{\frac{1}{2}} - \mu_{ij}^P\right] \qquad (t = T), (4.11)$$

where δ_{ij}^P is physical depreciation rate of corporate capital,
μ_{ij}^P is intercept parameter in Uzawa-Penrose function, and
η_{ij}^P is slope parameter in Uzawa-Penrose function.

Existence of the adjustment cost implies that rapid capital accumulation needs more capital installation cost, and as a result, desired levels of capital stock are attained gradually with instantaneous changes in the rate of return. Furthermore, incorporating adjustment cost in capital installation brings a positive meaning to an enterprise's optimal choice of investment. In cases where there is no adjustment cost, the model essentially solves an optimal accumulation path of capital stock so that the levels of investment in every period are derived in a passive manner. Its process is just equivalent to solving a static cost minimization problem by the production segment independently from the dynamic one. In contrast, the optimal levels of investment are determined first with the presence of adjustment cost, then capital is accumulated as a result. In consequence, an enterprise's expectation on the future economic condition affects its investment plan through the price of capital when there exists an adjustment cost, while a shock in any future period does not have any direct influence without the cost.

The production activity has a nested structure with constant returns to scale (CRTS) technologies such that:

$$Y_{ijt} = \Delta_{ij}^Y\left(\frac{K_{jt}^E}{\sum_{i'} Y_{i'jt}}\right)^{\omega_i^Y}\left\{\alpha_{ij}^Y\left(K_{ijt}^P\right)^{(\sigma_i^Y-1)/\sigma_i^Y} + \left(1-\alpha_{ij}^Y\right)L_{ijt}^{(\sigma_i^Y-1)/\sigma_i^Y}\right\}^{\sigma_i^Y/(\sigma_i^Y-1)} \qquad (4.12),$$

and

$$Q_{ijt} = \Delta_{ij}^{Q}\left\{\alpha_{ij}^{Q} Y_{ijt}^{\left(\sigma_i^Q-1\right)/\sigma_i^Q} + \left(1-\alpha_{ij}^{Q}\right)\hat{O}_{ijt}^{\left(\sigma_i^Q-1\right)/\sigma_i^Q}\right\}^{\sigma_i^Q/\left(\sigma_i^Q-1\right)} \tag{4.13}$$,

where Y_{ijt} is value added,
K_{jt}^{E} is stock of economic infrastructure,
ω_i^{Y} is shape parameter
σ_i^{Y} and σ_i^{Q} are elasticity of substitution,
α_{ij}^{Y} and α_{ij}^{Q} are share parameters, and
Δ_{ij}^{Y} and Δ_{ij}^{Q} are unit coefficients.

As in the case of social infrastructure in Equation (4.5), economic infrastructure K_{jt}^{E} is divided by the economy-wide amount of value-added $\sum_{i'} Y_{i'jt}$. We presume that the roads, bridges, ports, and so on, will be congested as economic activities increase. Productivity is enhanced through two channels. Relative increase of economic infrastructure over total value-added in the economy directly brings Hicks neutral type technical change, while relative increase of social infrastructure over total population of the economy indirectly affects production through Harrod neutral type labor-augmenting technical change.

As noted, investment and production segments chooses time paths of investment $\hat{F}_{ijt}^{P}$, gross output Q_{ijt}, intermediate input $\hat{O}_{ijt}$, and input of effective labor L_{ijt} to maximize the objective function shown as Equation (4.9) subject to Equations (4.10) through (4.13). The activity of the sales segment is explained in the next section.

4.4 Interregional Trade and Commodity Aggregators

In this section, we explain the transformation and aggregation of commodities produced in every region sold in both intra- and interregional markets. Commodities are assumed to be imperfect substitutes for that of another to handle cross-hauling, based on trade flows per dealer/merchant $E_{ijj't}$ from j'-th source region to j-th destination. As mentioned previously, the model is capable of flexibly choosing three kinds of specifications presented by Armington (1969), Krugman (1980), and Melitz

(2003) for Sector $i = 1$ that is assumed to exhibit IRTS, based on the supermodel developed by Dixon and Rimmer (2012) and its application by Oyamada (2013).

Assuming the existence of two kinds of fixed cost, one is necessary to establish a firm in a region ψ_j^K, and another is required to make sales on j-j' link $\psi_{jj'}^M$, gross output Q_{ijt} of production sector is transformed into trade flows per dealer/merchant $E_{ijj't}$ according to the following rule:

$$\sum_{j'}\left(1-\xi_{j'jt}\right)M_{jt}\frac{E_{ij'jt}}{\nabla_{j'jt}^M} = Q_{ijt}\left\{1-\sum_{j'}\left(1-\xi_{j'jt}\right)M_{jt}\psi_{j'j}^M - M_{jt}\psi_j^K\right\}, \qquad (4.14)$$

where M_{jt} is the number of dealers/merchants registered in j,
$\xi_{jj't}$ is the proportion of registered but inactive firms, and
$\nabla_{jj't}^M$ is average productivity of dealers/merchants making sales on j-j' link.

Then, two kinds of commodities from every region are aggregated according to the following two-stage nested function in a destination region to form intermediate, consumption, and capital goods:

$$\sum_{i'} O_{ii'jt} + C_{ijt}^P + C_{ijt}^G + F_{ijt}^P + F_{ijt}^E + F_{ijt}^S$$
$$= \Delta_{ij}^T\left\{\sum_{j'} \alpha_{ijj'}^T\left(1-\xi_{jj't}\right)M_{j't}\,E_{ijj't}^{(\sigma_i^T-1)/\sigma_i^T}\right\}^{\sigma_i^T/(\sigma_i^T-1)}, \qquad (4.15)$$

and

$$\hat{O}_{ijt} = \Delta_{ij}^O\left\{\sum_{i'} \alpha_{i'ij}^O\, O_{i'ijt}^{(\sigma_i^O-1)/\sigma_i^O}\right\}^{\sigma_i^O/(\sigma_i^O-1)}, \qquad (4.16)$$

where $O_{ii'jt}$ is regional composite of intermediate input,
C_{ijt}^P is regional composite for private consumption,
C_{ijt}^G is regional composite for government consumption,
F_{ijt}^P is regional composite for private gross fixed capital formation (GFCF),
F_{ijt}^E is regional composite for GFCF for economic infrastructure,
F_{ijt}^S is regional composite for GFCF for social infrastructure,
σ_i^T and σ_i^O are elasticity of substitution,

$\alpha^T_{ijj'}$ and $\alpha^O_{iji'}$ are share parameters, and
Δ^T_{ij} and Δ^O_{ij} are unit coefficients.

Equation (4.16) shows the case of sectoral composite for intermediate input $\hat{O}_{ijt}$. The cases for private consumption $\sum_s N_{jst}\,\hat{c}^P_{jst}$, government consumption $\hat{C}^G_{jt}$, private GFCF $\hat{F}^P_{jt}$, GFCF for economic infrastructure $\hat{F}^E_{jt}$, and GFCF for social infrastructure $\hat{F}^S_{jt}$ are all similar to the one expressed as Equation (4.16).

Then, relations between prices become:

$$\left(1+\tau^M_{ijj'}\right)\left(1+\tau^T_{ijj'}\right)p_{ijj't}$$

$$=\alpha^T_{ijj'}p^M_{ijt}\left(\Delta^T_{ij}\right)^{(\sigma^T_i-1)/\sigma^T_i}\left(\frac{\sum_{i'}O_{ii'jt}+C^P_{ijt}+C^G_{ijt}+F^P_{ijt}+F^E_{ijt}+F^S_{ijt}}{E_{ijj't}}\right)^{1/\sigma^T_i}, \qquad (4.17)$$

and

$$\left(1+\tau^O_{ii'j}\right)p^M_{ijt}=\alpha^O_{ii'j}p^O_{ijt}\left(\Delta^O_{i'j}\right)^{\left(\sigma^O_{i'}-1\right)/\sigma^O_{i'}}\left(\frac{\hat{O}_{i'jt}}{O_{ii'jt}}\right)^{1/\sigma^O_{i'}}, \qquad (4.18)$$

where $\tau^M_{ijj'}$ is import tariff rate, and
$\tau^O_{ii'j}$ is indirect tax rate on intermediate input.

The number of registered dealers/merchants M_{jt} is determined at the level that satisfies temporal profit becomes zero. That is given by:

$$\left\{\sum_{j'}\left(1-\xi_{j'jt}\right)\psi^M_{j'j}+\psi^K_j\right\}p^W_{ijt}Q_{ijt}=-\varepsilon\sum_{j'}\left(1-\xi_{j'jt}\right)p_{ij'jt}\,E_{ij'jt}, \qquad (4.19)$$

where $p_{ijj't}$ is markup price, and
ε is price markup rate such that $\varepsilon=-1/\sigma^T_i$.

The markup price $p_{ijj't}$ is determined by the price markup rule:

$$p_{ijj't} = \left(\frac{1}{1+\varepsilon}\right)\frac{p^W_{ij't}}{\nabla^M_{jj't}}. \tag{4.20}$$

The proportion of registered but inactive firms $\xi_{jj't}$ and the average productivity of active dealers/merchants operating on j-j' link $\nabla^M_{jj't}$ are given by:

$$\xi_{jj't} = 1 - \left(\frac{\zeta}{\zeta - \sigma^T_i + 1}\right)^{\zeta/(\sigma^T_i - 1)} \left(\nabla^M_{jj't}\right)^{-\zeta}, \tag{4.21}$$

and

$$\nabla^M_{jj't} = \left(\frac{\zeta}{\zeta - \sigma^T_i + 1}\right)^{\frac{1}{\sigma^T_i - 1}} \frac{(-\varepsilon)^{1/\left(1-\sigma^T_i\right)}}{1+\varepsilon} \left(\frac{p^W_{ij't}}{p_{ijj't}}\right)^{\sigma^T_i/(\sigma^T_i - 1)} \left(\frac{\psi^M_{jj'} Q_{ij't}}{E_{ijj't}}\right)^{1/(\sigma^T_i - 1)}, \tag{4.22}$$

where ζ is a Pareto shape parameter on productivity.

Finally, the switch between Melitz-type, Krugman-type, and Armington-type formulations is as follows. Set $\varepsilon = -\frac{1}{\sigma^T}$ to select a Melitz-type. Set $\psi^M_{j'j} = 0$, $\varepsilon = -\frac{1}{\sigma^T}$, $\nabla^M_{jj't} = 1$, and $\xi_{jj't} = 0$ to select a Krugman-type. Set $\psi^K_j = \psi^M_{j'j} = 0$, $\varepsilon = 0$, $\nabla^M_{jj't} = 1$, $\xi_{jj't} = 0$, and $M_{j't} = 1$ to select an Armington-type.

4.5 Government and Foreign Aid

In the model, the government in every region is assumed to stay passive. This assumption implies that it never makes any dynamic decision to maximize some objective. The reason is because if we assume an active government such that chooses, for instance, levels of taxes or volumes of public investment to maximize regional welfare, the model becomes AK type such as Barro (1990), which always remains in a steady state and does not show any transition. In other words, if a shock is given, the economy just jumps from a steady state to a new steady state. If it is the case, interesting features of a dynamic model may totally be lost. In this reason, we decided not to assume active government. Therefore, every public budget item is determined as a fixed proportion of the total budget.

In the economy, there are 15 kinds of taxes/subsidies. The revenues from those taxes minus subsidies form the base of fiscal budget. The total tax revenue $\mathit{\Gamma}_{jt}$ can be expressed as:

$$
\begin{aligned}
\mathit{\Gamma}_{jt} \equiv & \sum_{s=1}^{S} \left[\begin{array}{c}
N_{jst} \left[\begin{array}{c} \tau_j^A r_{jt} a_{jst} \\ +\tau_j^L \left(1-\phi_j^F-\phi_j^P\right) w_{jt}^L h_{jst} \left\{1-\chi_j\left(1-\upsilon_j\right) z_{jt} - f_{jst}^H\right\} \\ -\tau_j^Z w_{jt}^L h_{jst} \chi_j \left(1-\upsilon_j\right) z_{jt} \end{array} \right] \\
+N_{jst} \left\{ \begin{array}{c} \tau_j^B \left(1-\frac{\Omega_{js+1t+1}}{\Omega_{jst}}\right) b_{jt} \\ +\tau_j^A r_{jt} \left(a_{jst}+a_{jst}^F\right) \\ +\tau_j^L \left(1-\phi_j^F-\phi_j^P\right) w_{jt}^L h_{jst} \left(1-\chi_j \upsilon_j z_{jt-1}\right) \\ -\tau_j^Z w_{jt}^L h_{jst} \chi_j \upsilon_j z_{jt-1} \end{array} \right\} \\
+N_{jst} \left(\begin{array}{c} \left[\tau_j^B \left\{1+\left(1-\tau_j^A\right) r_{jt}\right\} + \tau_j^A r_{jt}\right] \left(1-\frac{\Omega_{js+1t+1}}{\Omega_{jst}}\right) b_{jt-1} \\ +\tau_j^A r_{jt} \left\{a_{jst}+\left(1-\tau_j^I\right) a_{jst}^F\right\} \\ +\tau_j^I \left(a_{jst}^F+\Lambda_{jt} a_{jst}^P\right) \end{array} \right) \\
+N_{jst} \left\{ \begin{array}{c} \left(\begin{array}{c} \tau_j^B \left\{1+\left(1-\tau_j^A\right) r_{jt}\right\} \left\{1+\left(1-\tau_j^A\right) r_{jt-1}\right\} \\ +\tau_j^A \left[r_{jt} \left\{1+\left(1-\tau_j^A\right) r_{jt-1}\right\} + r_{jt-1}\right] \end{array} \right) b_{jt-2} \\ +\tau_j^A r_{jt} a_{jst} \\ +\tau_j^I \Lambda_{jt} a_{jst}^P \end{array} \right\}
\end{array} \right] \\
& + \sum_i \left[\begin{array}{c} \tau_{ij}^V \left\{ \left(\frac{1}{1+\tau_{ij}^Q}\right) p_{ijt}^W Q_{ijt} - p_{ijt}^O \hat{O}_{ijt} - w_{jt}^L L_{ijt} \right\} \\ + \left(\frac{\tau_{ij}^Q}{1+\tau_{ij}^Q}\right) p_{ijt}^W Q_{ijt} - \tau_{ij}^F p_{jt}^P \hat{F}_{ijt}^P \end{array} \right] \\
& + \sum_i \left\{ p_{ijt}^M \left(\sum_{i'} \tau_{ii'j}^O O_{ii'jt} + \tau_{ij}^C C_{ijt}^P + \tau_{ij}^G C_{ijt}^G + \tau_{ij}^P F_{ijt}^P + \tau_{ij}^E F_{ijt}^E + \tau_{ij}^S F_{ijt}^S \right) \right\} \\
& + \sum_i \sum_{j'} \left[\tau_{ijj'}^M \left(1+\tau_{ijj'}^T\right) \left\{ p_{i=IRTSjj't} \left(1-\xi_{jj't}\right) M_{j't} + p_{i=CRTSjj't}^W \right\} E_{ijj't} \right],
\end{aligned}
\tag{4.23}
$$

where τ_{ij}^C is indirect tax rate on private consumption,

τ_{ij}^G is indirect tax rate on government consumption,

τ_{ij}^{P} is indirect tax rate on private GFCF,
τ_{ij}^{E} is indirect tax rate on GFCF for economic infrastructure, and
τ_{ij}^{S} is indirect tax rate on GFCF for social infrastructure.

The foreign aid receipt for general budget support can be set as:

$$\sum_{j'}\left(1-\Xi_{jj't}^{E}-\Xi_{jj't}^{S}\right)D_{jj't}, \tag{4.24}$$

where $D_{jj't}$ is foreign aid flow from j'-th donor to j-th recipient,
$\Xi_{jj't}^{E}$ is the proportion tied to GFCF for economic infrastructure, and
$\Xi_{jj't}^{S}$ is the proportion tied to GFCF for social infrastructure.

Interest and repayment of foreign aid loans paid to j'-th donor is:

$$\sum_{j'}\left\{\begin{matrix}\left(\Xi_{jj't-1}^{1}+r_{jj't-1}^{A}\right)\left(1-\Xi_{jj'}^{D}\right)D_{jj't-1}\\ +\left(1+r_{jj't-2}^{A}\right)\left(1-\Xi_{jj't-2}^{1}\right)\left(1-\Xi_{jj'}^{D}\right)D_{jj't-2}\end{matrix}\right\}, \tag{4.25}$$

where $r_{jj't}^{A}$ is the lending rate of foreign aid loans determined by a contract,
$\Xi_{jj'}^{D}$ is grant element of foreign aid funds, and
$\Xi_{jj't}^{1}$ is the share of foreign aid loans required to be repaid in the first period.

Then, overall fiscal budget Θ_{jt} can be expressed as:

$$\Theta_{jt}\equiv\left(1-\Xi_{j}^{G}\right)\left[\begin{matrix}\Gamma_{jt}\\ +\sum_{j'}\left(1-\Xi_{jj't}^{E}-\Xi_{jj't}^{S}\right)D_{jj't}\\ +\sum_{j'}\left\{\begin{matrix}\left(\Xi_{j'jt-1}^{1}+r_{j'jt-1}^{A}\right)\left(1-\Xi_{j'j}^{D}\right)D_{j'jt-1}\\ +\left(1+r_{j'jt-2}^{A}\right)\left(1-\Xi_{j'jt-2}^{1}\right)\left(1-\Xi_{j'j}^{D}\right)D_{j'jt-2}\end{matrix}\right\}\\ -\sum_{j'}\left\{\begin{matrix}\left(\Xi_{jj't-1}^{1}+r_{jj't-1}^{A}\right)\left(1-\Xi_{jj'}^{D}\right)D_{jj't-1}\\ +\left(1+r_{jj't-2}^{A}\right)\left(1-\Xi_{jj't-2}^{1}\right)\left(1-\Xi_{jj'}^{D}\right)D_{jj't-2}\end{matrix}\right\}\end{matrix}\right], \tag{4.26}$$

where $\varXi_j^G$ is the government saving rate (fixed at this stage).

Note that $\varXi_j^G$ tend to be negative in the model to generate fiscal deficit. Fiscal deficit is financed by issues of government securities. We presume the government securities take a form of one period bond, which is redeemed at the unity price.

Let us move to the expenditure side. There are five expenditure items determined as fixed proportions of the fiscal budget $\varTheta_{jt}$. They are foreign aid disbursement $\varXi_j^A \varTheta_{jt}$, compensation for PAYG fund $\varXi_j^P \varTheta_{jt}$, government consumption $\varXi_j^C \varTheta_{jt}$, and public investment to two kinds of infrastructure. The budgets for two kinds of public investment are noted as:

$$\varXi_j^F \left(1 - \varXi_j^A - \varXi_j^P - \varXi_j^C\right)\varTheta_{jt} + \sum_{j'} \varXi_{jj't}^E \left(1 - \varXi_{jj'}^D\right) D_{jj't}, \tag{4.27}$$

and

$$\left(1 - \varXi_j^F\right)\left(1 - \varXi_j^A - \varXi_j^P - \varXi_j^C\right)\varTheta_{jt} + \sum_{j'} \varXi_{jj't}^S \left(1 - \varXi_{jj'}^D\right) D_{jj't}, \tag{4.28}$$

where $\varXi_j^F$ is the proportion of economic infrastructure in public investment.

The second terms in Equations (4.27) and (4.28) correspond to foreign aid disbursements respectively tied to economic and social infrastructure.

The sovereign debt position G_{jt} is expressed as:

$$G_{jt} = \begin{cases} \bar{G}_{j0} \qquad (\bar{G}_{j0}\text{: given}) & (t = 0) \\ +G_{jt-1} - \left(\frac{\varXi_j^G}{1-\varXi_j^G}\right)\varTheta_{jt-1} & (t \neq 0), \end{cases} \tag{4.29}$$

and

$$\hat{\gamma}_j G_{jT} = -\left(\frac{\varXi_j^G}{1-\varXi_j^G}\right)\varTheta_{jT} \qquad (t = T). \tag{4.30}$$

We presume $\varXi_j^G \leq 0$ in a steady state.

Finally, the accumulations of two kinds of public capital are similar to the case of corporate capital.

4.6 Financial Portfolio

As noted before, the model presumes imperfectly substituting financial instruments to capture frictions in interregional capital movements from capital redundant aged region to labor redundant young region. In this section, we explain how the intra- and interregional capital movements are modeled.

Similar to the production part, we assume multi-stage portfolio using constant elasticity of transformation (CET) functions, following Rosensweig and Taylor (1990). While Rosensweig and Taylor (1990) utilizes constant elasticity of substitution (CES) functions to aggregate expected rate of return, most of the FOCs become identical with the ones that will be shown here.

Every investment trust bank operating in each region collects regular assets, FF pension reserves, and funds deposited to bequest account from individuals and invest the fund by proxy to every local asset markets beyond regional boundary. Note that the investment trust banks do not charge commissions since the model does not have a banking sector at this stage. In every asset market, financial instruments such as corporate capital and government securities have their own rates of return that are evaluated with risk premiums.

At the first stage, an investment trust decides portfolio among regions to maximize the return from instrumental composite of assets $A^M_{jj't}$. It is expressed as follows:

$$\max \quad \sum_j \frac{r^M_{jj't}}{1+\pi^G_{jt}} A^M_{jj't}$$

$$\text{s.t.} \quad \nabla^A_{j'} \left\{ \sum_j \alpha^A_{jj'} \left(A^M_{jj't} \right)^{(\sigma^A - 1)/\sigma^A} \right\}^{\sigma^A/(\sigma^A - 1)} = A^T_{j't}, \tag{4.31}$$

where $A^T_{j't}$ is the total assets collected from individuals,
π^G_{jt} is regional risk,
σ^A is elasticity of transformation,

$\alpha^{A}_{jj'}$ is share parameter, and
$\nabla^{A}_{j'}$ is unit coefficient.

The regional risk π^{G}_{jt} is defined by:

$$\pi^{G}_{jt} = \Delta^{R}_{j}\left\{\exp\left(\frac{G_{jt}}{\sum_i p^{Y}_{ijt}Y_{ijt}}\right) - 1\right\} + \pi^{B}_{j}, \tag{4.32}$$

where π^{B}_{j} is basic risk, and
Δ^{R}_{j} is unit coefficient.

The exogenously given basic risk π^{B}_{j} includes several elements of regional risk, such as political risk, conditional status of local capital market, and so on. The first term in the right-hand-side of Equation (4.32) scoops up the sovereign risk, which is endogenously determined by the level of sovereign debt position over GDP.

The second choice of an investment trust is portfolio between financial instruments, i.e., government securities $A^{G}_{jj't}$ and sectoral composite of corporate capital $A^{B}_{jj't}$ to maximize the return from both kinds of asset. The problem is:

$$\begin{aligned} \max \quad & r^{G}_{jt}A^{G}_{jj't} + \frac{r^{B}_{jj't}}{1+\pi^{K}_{j}}A^{B}_{jj't} \\ \text{s.t.} \quad & \nabla^{G}_{jj'}\left\{\alpha^{G}_{jj'}\left(A^{G}_{jj't}\right)^{(\sigma^{G}-1)/\sigma^{G}} + \left(1-\alpha^{G}_{jj'}\right)\left(A^{B}_{jj't}\right)^{(\sigma^{G}-1)/\sigma^{G}}\right\}^{\sigma^{G}/(\sigma^{G}-1)} = A^{M}_{jj't}, \end{aligned} \tag{4.33}$$

where r^{G}_{jt} is the rate of return on government securities,
$r^{B}_{jj't}$ is the rate of return on sectoral composite of corporate capital, and
π^{K}_{j} is instrumental risk of the corporate capital.

An investment trust bank's final choice is sectoral portfolio among corporate capital to maximize the total return from every corporate capital $A^{K}_{ijj't}$. The problem is:

$$\max \quad \sum_i \frac{r_{ijt}^K}{1+\pi_{ij}^S} A_{ijj't}^K$$

$$\text{s.t.} \quad \nabla_{jj'}^K \left\{ \sum_i \alpha_{ijj'}^K \left(A_{ijj't}^K \right)^{(\sigma^K-1)/\sigma^K} \right\}^{\sigma^K/(\sigma^K-1)} = A_{jj't}^B, \tag{4.34}$$

where r_{ijt}^K is the rate of return on corporate capital, and
π_{ij}^S is sectoral risk.

For the example of the sectoral risk π_{ij}^S, unsettled weather for agricultural sector, changes in the market environment, and so on, can be listed.

4.7 Market Equilibrium

In this section, we will see the equilibrium conditions to close the model.

First, the following condition must hold for the commodity market:

$$\sum_{i'} O_{ii'jt} + C_{ijt}^P + C_{ijt}^G + F_{ijt}^P + F_{ijt}^E + F_{ijt}^S$$
$$= \Delta_{ij}^T \left\{ \sum_{j'} \alpha_{ijj'}^T \left(1 - \xi_{jj't}\right) M_{j't} E_{ijj't}^{(\sigma_i^T-1)/\sigma_i^T} \right\}^{\sigma_i^T/(\sigma_i^T-1)}. \tag{4.35}$$

Second, the market clearing condition for the effective labor is:

$$L_{ijt} = \sum_{s=1}^{2} \left(N_{jst} h_{jst} \left[1 - \left\{ \chi_j \left(1 - \upsilon_j\right) z_{jt} + f_{jst}^H \right\} - \chi_j \upsilon_j z_{jt-1} \right] \right). \tag{4.36}$$

Third, we set the equilibrium condition for the corporate capital as:

$$\left(\frac{1}{1+r_{ijt}^K} \right) p_{ijt}^K K_{ijt}^P = \sum_{j'} A_{ijj't}^K. \tag{4.37}$$

Fourth, the market clearing condition for the government securities can be expressed as:

$$\left(\frac{1}{1+r_{jt}^G} \right) G_{jt} = \sum_{j'} A_{jj't}^G, \tag{4.38}$$

where the price (nominal par) of government securities is set to unity. The government securities are assumed to take the form of one period bond, and their market price is defined by its temporal rate of return r_{jt}^G.

By the Walrus law, one of the above equilibrium conditions automatically holds. Therefore we drop a condition giving a price or a rate of return exogenously, while we have not yet decided which is the one, at this stage.

Finally, we need equilibrium conditions with respect to time t. At the terminal period $t = T$, economies must be in a steady state. In a steady state, all quantity variables grow at the same overall growth rate $\hat{\gamma}_j$, which is determined endogenously, while all price variables stay at constant levels. Those conditions are given as Equations (4.3) and (4.11), and the relations corresponding to $t = T$ in FOCs for dynamic problems.

4.8 Concluding Remarks

In this chapter, the basic structure and the major assumptions of the three-region, two-sector OLG/AGE model developed for this research project are reported. It can be said that our challenges in building the model reached quite an ambitious level. We believe a model that includes this level of elements cannot so easily be found.

However, there still are potentially important elements that we have not been taken into account in our framework. One example is the problem of migration and remittances. As capital that flows beyond regional boundary, people moves from one region to another pursuing jobs and higher salaries. The gaps between capital and labor in both young and aged regions can be filled not only by interregional capital movements nor foreign aids, but also by migrations. The reason why we have not yet succeeded to include migration and remittance into the model is because it will make the structure of an individual’s budget constraint too complicated to handle. One may notice that Equations (4.2) and (4.3), for instance, have already reached a “too much” level. Since we are handling the effective labor that enhances productivity based on one’s choice of schooling, moving an individual beyond regional boundary tracing his/her home, career of schooling, and so on, increases dimensions of a model to reach a non-manageable level. One more reason is that even a small demographic shock may bring crucial impact to economies in a model. Therefore, an inclusion of migration makes a model quite difficult to solve. It must be a real challenge.

Another example is negative bequest, i.e., children who help their parents. The reasons why parents have many children in developing economies are regarded that poorness of pension system and parents' expectations for the support provided by their children. The model might be modified and extended by including utility from supporting a parent.

Since nothing is more complex than the real economy, people will never satisfy the volume of an analytical model as avarice knows no bounds. Let us continue working through various simulation analyses to bring the model to perfection some day. On the other hand, unfortunately, incorporating a complex structure increases requirements on data. Basically, a benchmark data set has to satisfy all of the constraints included in a model. Otherwise, data is adjusted in reconciliation works. In the next chapter, we will see such problems we found through this model building work.