Chapter 2. Commodity Futures Market

Tsubasa Shibata and Hiroyuki Kosaka

1. Introduction

Changes in crude oil price affect inflation, output, and economic growth. The development of derivative markets makes it difficult to predict commodity prices. Prediction of the crude oil price plays an important role in conducting monetary policy. Therefore, it has been increasingly importance to clarify the price determination mechanism of futures prices and spot prices and forecast as accurately as possible (Greenspan, 2004, 2005; Bernanke 2004, 2006, Kohn, 2007)

In this interim report, we provide an overview of modeling approaches for commodity future market by shedding light on the relation between the spot price and the futures price and Futures transactions are part of the derivative financial instruments which include forward, options and swaps, but this paper focuses only on futures market.

Specifically, the rest of this paper is composed of four parts. Section 2 refers some representative the model for future commodity market. In particular, we cover the important models: a model based on the theory the Cost-of-Carry, a model based on the Risk Premium such as Capital Asset Pricing Model (CAPM), and the Black and Scholes Model (BS), and Neural Network etc. We also show some empirical estimation results. Finally, Section 4 concludes this paper.

2. Model for Commodity Future Market

2.1. Futures Price Determination I

- Modeling based on Perspective of Financial Instruments-

This sub-section sheds light on market participants such as financial institution and speculators who do not intend to trade physical commodities and considers how the futures price determines there. The first two models were developed as theories that determine prices of risky securities based on the financaila theory. We think that they can apply for pricing of future commodities.

(1) Capital Asset Pricing Model (CAPM)

CAPM (Capital Asset Pricing Model) is a model that contributes to explain how financial markets price risky securities. The theory of CAPM intends to describe optimal portfolio between risk-free asset and risky assets. This model assumes that the expected rate of return on the asset is a linear proportion of the market portfolio. Namely, it means that the expected return on the asset can be expressed by a linear function of the expected rate of return of the market such as TOPIX. In this sub-section, we trace CAPM model below.

It is assumed the equilibrium of the market portfolio and consider its equilibrium return rate. Here, we consider portfolio p which consists of combination with w portion invested in an asset and $(1-w)$ portion invested in the market portfolio M. The return rate of this portfolio is as:

$$r_p = wr_0 + (1-w)r_M \tag{3.1}$$

Since $E(r_0) = \mu_0$ and $E(r_M) = \mu_M$, the expected rate of return can be written as follows;

$$\mu_p = E(r_p) = w\mu_0 + (1-w)\mu_M \tag{3.2}$$

In addition, its variance is as:

$$\begin{aligned} \sigma_p^2 &= E\left(r_p - \mu_p\right)^2 = E\left(w(r_0-\mu_0) + (1-w)(r_M-\mu_M)\right)^2 \\ &= w^2E(r_0-\mu_0)^2 + (1-w)^2E(r_M-\mu_M)^2 \\ &\quad + 2w(1-w)E(r_0-\mu_0)(r_M-\mu_M) \\ &= w^2\sigma_0^2 + (1-w)^2\sigma_M^2 + 2\mathrm{w}(1-w)\sigma_{0M} \end{aligned} \tag{3.3}$$

Here, the slope in the volatility- mean plane can be rewritten as:

$$\frac{d\mu_p}{d\sigma_p} = \frac{d\mu_p}{dw}\frac{dw}{d\sigma_p} = \frac{d\mu_p}{dw}\frac{1}{d\left(\sigma_p/dw\right)} = (\mu_0-\mu_M)\frac{\sigma_M}{\sigma_{0M}-\sigma_M^2} \tag{3.4}$$

where σ_{0M} is the covariance between the rate of return of an asset 0 and the rate of return of the market portfolio. The slope of capital market line means the sharp ratio of the market portfolio $\left(\mu_M - \mu_f\right)/\sigma_M$ as:

$$\frac{d\mu_p}{d\sigma_p} = \frac{d\mu_p}{dw}\frac{dw}{d\sigma_p} = (\mu_0 - \mu_M)\frac{\sigma_M}{\sigma_{0M} - \sigma_M^2} = \frac{\mu_M - \mu_f}{\sigma_M} \tag{3.5}$$

By rearranging equation (3.5), we can get the following equation.

$$\begin{aligned} \mu_0 &= \mu_M + (\mu_M - \mu_f)\left(\frac{\sigma_{0M}}{\sigma_M^2} - 1\right) = \mu_M + (\mu_M - \mu_f)\left(\frac{\sigma_{0M}}{\sigma_M^2}\right) - (\mu_M - \mu_f) \\ &= \mu_f + \left(\frac{\sigma_{0M}}{\sigma_M^2}\right)(\mu_M - \mu_f) = \mu_f + \beta(\mu_M - \mu_f) \end{aligned} \tag{3.6}$$

This is the traditional framework of CAPM which provide us with theoretically appropriate required rate of return of an asset.

Here, we assume that return rates of assets would change over time. In order to reflect that, we put a disturbance term u_t in its relation for asset *i* at time *t* as,

$$r_{0,t} - r_{f,t} = \beta(\mu_{M,t} - \mu_{f,t}) + u_t \tag{3.7}$$

where the term $\beta = \sigma_{0M}/\sigma_M^2$. β means the volatility of an asset in relation to the overall market. An implication of β as a regression coefficient show return of a stock changes towards changes the markets as a whole, namely relative risk of the asset.

Here, we can apply the CAPM where a return rate of futures price as a dependent variable is explained by a return rate of market r_M and a safe asset such as a long-term government bond yield μ_f.

$$r_{com} - r_f = \beta(r_M - \mu_f) + u \tag{3.8}$$

where r_{com} is commodity futures price return and r_M is return on market portfolio, and r_f is U.S. Treasuries. Hence, the determination of American stock market index Commodity futures prices in Chicago Mercantile Exchange (CME) depends upon Standard & Poor's 500 Stock Index (SP500) and 10 Year US government bond yield.

(2) Arbitrage Pricing Theory (APT)

Arbitrage Pricing Theory (APT) was developed by S.Ross (1976) as an alternative to the CAMP. The ATP model assumes that return of assets return *r* is formulated as follows:

$$r = a + \sum_{i=1}^{K} b_i F_i + u \qquad (3.9)$$

Thus, the APT is a generalized CAPM model.

$$r_{com} - r_f = \beta(r_M - r_f) + \sum_{i=1}^{m} \alpha_i Z_i + u \qquad (3.10)$$

(3) Neural Network

Other approaches have been developed to make more accurate predictions about futures prices then CAPM. Notably, Takami (2006) attempts to apply the neural network to forecast futures prices by using data of 18 future contracts traded on the Tokyo Commodity Exchange (TOCOM). This model is composed of three layers, called the input layer, hidden layer, and output layer. The hidden layer of three elements is described by a nonlinear mathematical function called a sigmoid.

However, since parameters are not explicit, it is unacceptable to utilize economic modeling.

(4) Kuchiki and Ogawa Model (1990)

Kuchiki and Ogawa (1990) predicts the real spot price by its futures price which is formed at *t*-3 term. A spot price is explained by its futures price which is a prior.

$$p_t^s = a_{t-3} + \left(\frac{1}{U_{t-3}}\right) p_{t,t-3}^f + e_t \qquad (3.11)$$

p_t^s is spot price at *t*-term and $p_{t,t-3}^f$ is its futures price which is formed at (*t*-*3*)-term. By solving equation (3.11) for $p_{t,t-3}^f$, the commodity futures price is derived as:

$$p_{t,t-3}^f = a'_{t-3} + U_{t-3} p_t^s - U_{t-3} e_t \qquad (3.12)$$

Coefficients in equation (3.12) can be estimated by employing Kalman filter, which enables to calculate time varying parameters. Here, instead of an approach by Kalman filter, we introduce the sigmoid activation function, which is traditionally a popular activation function for neural networks. The following function is yield.

$$p_{t,t-3}^{f} = a_0 + a_1 \left(\frac{1}{\left(1 + e^{-@pch(p_t^s)}\right)} \right) p_t^s \tag{3.13}$$

In equation (3.13), there is a feature that its parameter of the sigmoid activation function shifts in going up or down.

2.2. Futures Price Determination II

- Modeling based on an extension of Spot Price –

This subsection illustrates models in which the futures price is explained in the context of spot price. Namely, they have the framework that the commodity spot prices exist a priori and thereby the futures prices are determined. Specifically, it is supposed that we traders who deal with physical commodities attempt to forecast the futures prices based on the current transaction. In this approach, it is imperative to take into account a connection between spot mark and futures mark model.

(1) Futures Curve Model – Term Structure of Futures Prices

Firstly, we begin with introducing futures curve model as a fundamental model that explore to explain the futures commodity price. Futures curve provide us with valuable information how market participants anticipate price movement. The futures curve is the current price for a commodity at specified data in the future. The contract expiration dates of a commodity is plotted along an X-axis and futures prices is shown along a Y-axis. There are three-year and five-year future contracts in contract expiration dates. Although the futures curve is not a main indicator for determining this future market trend, it is useful to understand its shape to make appropriate trading decision.

Investors and speculators are concerned with the shape of futures curve, namely futures price movement: contango or backwardation. If the future curve is upward sloping as time moves, it implies contango. It implies that futures price of a commodity is over the expected spot price. A contango marketis knows as a normal market (a positive spread) in which further contract is more expensive than the nearby contract. Specifically, precious metals such as gold, platinum, and silver usually tends to be contango. The opposite of contango is backwardation. When the future curve is downward over time, it means backwardation.

Besides simple contango and backwardation, there are futures curve which reflect seasonal fluctuations such as agricultural and energy commodity. For example, the futures curve of agriculture commodities such as corn, soybean, and wheat has the seasonal patterns for planting, harvesting, and marketing. In the harvest season, the increase of expected supply leads to backwardation in which the futures price is lower than the spot price. As for energy commodity, the futures curves such as natural gas and heating oil display contango in the winter season in which their futures price is higher than the spot price because of the increase of the expected supply.

What factors determine the shape of the futures curve? The Futures price is defined as: *The Futures Price = Spot Price + Finance + Storage Cost -Convenience Yield*. The finance means interest rate on the money borrowed to own the physical commodity. Convenience yield is a measure of the benefits from ownership of an asset that are not obtained by the holder of a long futures contract on the asset (Hull, 2015).

Now, we consider that investor purchase a physical commodity in current spot price S_t at time *t* and deliver future contract simultaneously at the delivery date *T*. the futures price is given by.

$$F_t^T = S_t e^{(r+c-\varepsilon)(T-t)} \tag{3.14}$$

where r is zero-coupon risk-free rate of interest, c is the storage cost, and ε is the convenience yield. At $t = T$, we can obtain $F_T^T = S_T e^{(r+c-\varepsilon)(T-T)} = S_T$. We can refer the cost of storing a physical commodity as the cost of carry that summarizes financial costs such as interest costs r, the storage cost c, and the convenience yield ε. The futures price depend upon the current spot price and the cost of carry. As equation (3.14) tells, if the convenience yield ε is higher, the futures price F_t^T falls.

(2) <u>Extending Capital Assets Pricing Model by K.Dusak(1973)</u>

Dusak (1973) examined the existence of risk premium of wheat, corn and soybean futures from data which covered semimonthly the period 1952 to 1967 by applying the theory of classic CAPM. The classic theory CAPM begins at $\mu_0 - \mu_f = \beta(\mu_M - \mu_f)$.we assume that the return on any capital asset *i*, is represented as:

$$E(R_i) = (1 - \beta_i)\mu_f + \beta_i \mu_M \tag{3.15}$$

where R_i is the return rate on asset *i* and $E(R_i)$ is its expectation. $E(R_i)$ is defined,

$$E(R_i) = \frac{E(P_{i,1}) - P_{i,0}}{P_{i,0}} \tag{3.16}$$

By inserting $E(R_i)$ of equation (3.16) into equation (3.15), we can obtain the following equation.

$$\frac{E(P_{i,1}) - P_{i,0}}{P_{i,0}} = (1 - \beta_i)\mu_f + \beta_i \mu_M \tag{3.17}$$

Rearranging equation (3.17), the following equation is derived.

$$\frac{E(P_{i,1})}{P_{i,0}} = (1 - \beta_i)\mu_f + \beta_i \mu_M + 1 \tag{3.18}$$

Repeating the same process, we obtain as:

$$\begin{aligned} E(P_{i,1}) &= (1 - \beta_i)\mu_f P_{i,0} + \beta_i \mu_M P_{i,0} + P_{i,0} \\ &= (1 + \mu_f)P_{i,0} - \beta_i \mu_f P_{i,0} + \beta_i \mu_M P_{i,0} \\ &= (1 + \mu_f)P_{i,0} + (\mu_M - \mu_f)\beta_i P_{i,0} \end{aligned} \tag{3.19}$$

Moreover, by arranging equation (19), we can rewrite (19) as,

$$E(P_{i,1}) - (\mu_M - \mu_f)\beta_i P_{i,0} = (1 + \mu_f)P_{i,0} \tag{3.20}$$

or equivalently as

$$P_{i,0} = \frac{E(P_{i,1}) - (\mu_M - \mu_f)\beta_i P_{i,0}}{(1 + \mu_f)} \tag{3.21}$$

Multiplying both sides of equation (3.21) by $(1+\mu_f)$, the following equation is gained.

$$P_{i,0}(1+\mu_f) = E(P_{i,1}) - (\mu_M - \mu_f)\beta_i P_{i,0} \tag{3.22}$$

Here, the following relation is set as:

$$P_{f,0} = P_{i,0}(1+\mu_f) \tag{3.23}$$

The expression $P_{i,0}(1+\mu_f)$ means payment of the spot commodity one period later and the current futures price for delivery (Dusak, 1973). By substituting equation (3.23) for (3.22) and rearranging terms, we see that,

$$P_{f,0} = E(P_{i,0}) - (\mu_M - \mu_f)\beta_i P_{i,0} \tag{3.24}$$

We can rewrite equation (3.24) as follows:

$$\frac{E(P_{i,1}) - P_{f,0}}{P_{i,0}} = \beta_i(\mu_M - \mu_f) \tag{3.25}$$

Here, we assume a regression model of Dusak' model as,

$$\frac{p_t^f - p_{t-1}^f}{p_{t-1}} = \beta(\mu_M - \mu_f) \tag{3.26}$$

Furthermore, a regression model for approximation of Dusak' model is defined as,

$$\frac{p_t^f - p_{t-1}^f}{p_{t-1}^f} = \beta(\mu_M - \mu_f) \tag{3.27}$$

It is important that there is no μ_f in the left side of regression equation (3.27).
We attempt to estimate crude oil (WTI), cooper, and wheat model by utilizing data S&P 500 about μ_f. Table 3.1 shows estimation results. However, the calculation are not acceptable. We can see that it is difficult to apply Dusak model to commodity futures price directly.

Table 3.1. Estimation Results of Commodity Futures Price by Dusak Model

Commodities	β	*Adj. R*-squared
Crude Oil (WTI)	0.698	0.107
Copper	0.790	0.205
wheat	0.538	0.046

(3) Black-Scholes Model by Schwartz (1997)

There are a number of studies that aim to illustrate the term structure of commodity futures price which changes its futures curve. Schwartz (1997) employs the Black-Scholes (BS) Model about the volatility of physical asset S_t, based on futures curve in equation (3.14)

The first factor in this model correspond to introduction of BS model to the volatility of physical asset S_t. Additionally, as the second and the third factor, BS model is employed to explain the convenience yield of the commodity $\delta = (\varepsilon - c)$ and interest rate r. Schwartz (1997) two-facto model that the dynamics of the commodity spot price and convenience yield are given by the following stochastic process as:

$$dS = (\mu - \delta)Sdt + \sigma_1 Sdz^1 \tag{3.28}$$

$$d\delta = \kappa(\alpha - \delta)dt + \sigma_2 dz^2 \tag{3.29}$$

where dz^1 and dz^2 are Brownian motion. S is the spot price and δ is the convenience yield. The log form of the futures price with maturity *T* at time *t* is shown as:

$$\log F_t^T = \log S - \delta \frac{1 - e^{-\kappa T}}{\kappa} + A(T) \tag{3.30}$$

where

$$A(T) = \left(r - \hat{\alpha} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2} - \frac{\sigma_1\sigma_2\rho}{\kappa}\right)T + \frac{1}{4}\sigma_2^2\frac{1 - e^{-2\kappa T}}{\kappa^3} + \left(\hat{\alpha}\kappa + \sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa}\right)\frac{1 - e^{-\kappa T}}{\kappa} \tag{3.31}$$

Schwartz (1997) estimated this model for five futures contracts by using data those weekly observations of futures prices for two commodities, oil and cooper, and one precious metal, gold.

Iihara, Kato, and Tokunaga (2000) employed state-space model and examined Japan futures price for gold by applying Black and Scholes model based on Schwartz (1997, 1998). However, BS model has a drawback that it cannot explain the relation between futures prices movement and other factors such as macroeconomic and supply-demand for crude oil (e.g. OPEC supply cuts). In this sense, Black and Scholes model is no more than approximating such as Fourier series or Taylor expansion critically. Therefore, we can simplify $A(T)$ in equation (3.31) by applying our model the spot price determination (See Chapter 1 by Shibata and Kosaka) as:

$$A(T) = \left(r - \hat{\alpha} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2}\right)T + \frac{1}{4}\sigma_2^2\frac{1 - e^{-2\kappa T}}{\kappa^3} + \left(\hat{\alpha}\kappa - \frac{\sigma_2^2}{\kappa}\right)\frac{1 - e^{-\kappa T}}{\kappa^2} \tag{3.32}$$

(4) <u>Deep Learning for Time Series</u>

As we discussed in the futures curves, backwardation is normal market situation except the special commodities such as the precious metals and the seasonal commodities. The futures price eventually converges on the spot price. Assuming that, the gaps between the futures price and the spot price in situation of backwardation close as contract expiration come near. Hence, we can describe it as follow:

$$p_t^f = \gamma_0 + \gamma_1 p_{t-1} + \gamma_2\left(p_{t-1}^f - p_{t-1}\right) \tag{3.33}$$

or

$$p_t^f = \gamma_0 + \gamma_1 p_{t-1} + \gamma_2\left(\frac{p_{t-1}^f}{p_{t-1}}\right) \tag{3.34}$$

Expression of equation (3.34) implies that the futures price formation depends on two factors: the spot price at the nearest precious time and the difference between the futures price and the spot price.

Now, we assume that the futures curves shown in equation (3.14) is unidentified. Also, The spread between the futures price and the spot price at the time *t*-1 is supposed to hold at the current time *t* as follows:

$$p_t^f - p_t = p_{t-1}^f - p_{t-1} \tag{3.35}$$

where p_t^f denotes the futures price that is formed at the time *t*-1.

Modification 1

We take into consideration the relative relation both sides of equation (3.35) as follows:

$$p_t^f - p_t = \alpha_0 + \alpha_1\left(p_{t-1}^f - p_{t-1}\right) \tag{3.36}$$

Modification 2

Additionally, we assume that the spot price is unidentified. Reflecting that, we can rewrite equation (3.36) as:

$$p_t^f = \hat{p}_t + \alpha_0 + \alpha_1\left(p_{t-1}^f - p_{t-1}\right) \tag{3.37}$$

We assume that the prediction of the spot price follows,

$$\hat{p}_t = \beta_0 + \beta_1 p_{t-1} \tag{3.38}$$

Inserting (3.38) into (3.27) and rearranging it, the following model is obtained.

$$p_t^f = (\alpha_0 + \beta_0) + \beta_1 p_{t-1} + \alpha_1\left(p_{t-1}^f - p_{t-1}\right) \tag{3.39}$$

Table 3 illustrates the empirical results of commodity futures prices as crude oil, cooper, and wheat by employing the approach of deep learning for time series. The results tells that the spread between the futures price and the current price at previous term shows negative sign. While the model structure is so simple, it seem to be well estimated.

Table 3.2. Estimation Results of Futures Price by Deep Learning for Time Series

Commodities	γ_0	γ_1	γ_2	*Adj. R*-squared
Crude Oil	-13.779	1.464	-0.004	0.107
Copper	0.452	0.901	-460.139	0.205
wheat	352.285	0.650	-64.493	0.046

3. Conclusion

In this paper, we represent their model specifications of commodity futures mark: a model based on the theory the Cost-of-Carry, a model based on the Risk Premium such as Capital Asset Pricing Model (CAPM), and the Black and Scholes Model (BS), and Neural Network etc.

However, estimation results we conducted will require reconsideration for model sophistication. Additionally, we should cover options and swaps transactions in derivatives, which would enable to reflect the real economy.

These improvements will be implemented in future works.

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