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Chapter 2

Understanding Krugman's "Third-Generation" Model of Currency and Financial Crises

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Abstract

I construct a simplified but complete version of Krugman (1999) model, derive a closed-form solution, and make sure that there are two dynamic equilibria, one of which is the steady-growth one and the other of which is the currency-crisis one accompanied by balance-sheets crisis as Krugman suggested. I examine conditions for the existence of self-fulfilling crisis equilibrium and find that an economy may be faced with such crises if (1) propensity to import is low, (2) propensity to consume is low, (3) world interest rate is low, (4) borrowing constraint of private sector is moderate, (5) financial market is restrictive and there is high entry barrier, (6) price elasticity of export is low, and (7) wage elasticity of labor supply is low. I also show how the excess liabilities cause a large but temporal depreciation of exchange rate inevitably in the model.

Tightening monetary policy aiming to avoid a sharp depreciation of nominal exchange rate causes a deflation and makes the balance-sheet problem worse. Expansionary fiscal policy can avoid the crisis only if the government raises a sufficient fund from abroad, but the required amount of fund may be extraordinary large if the borrowing constraint is moderate.

Keywords: currency crisis, balance sheets, real exchange rate.

1. Introduction

Krugman (1999) presented a new theory of currency crisis, so-called a "third-generation" crisis model, which is different in various aspects from other currency-crisis models. In both "first-generation" models such as Krugman (1979) and Flood and Garber (1984) and "second-generation" models such as Obstfeld (1996), purchasing power parity (PPP) was assumed to holds, so that the sharp depreciation of nominal exchange rate in crisis was caused by inflation and a completely monetary phenomenon. On the other hand, there is no money explicitly in Krugman's model, and the depreciation of exchange rate is a fall of relative price of domestic product, which is a totally non-monetary phenomenon. Some "third-generation" models are also monetary ones. Aghion, Bacchetta, and Banerjee (2004) presented a model where the depreciation of currency today reduces production and causes inflation in future, and such an expectation of future inflation causes current depreciation of exchange rate. Chang and Velasco (2000) shows how the role of lender of last resort conflicts with the fixed rate policy, though it does not discuss the depreciation of exchange rate. Krugman himself didn't examine the model completely, and the discussion about the non-monetary mechanism of currency crisis is insufficient. Are such crises important or trivial?

I construct a simplified but complete version of Krugman (1999) model, derive a closed-form solution, and make sure that there are two dynamic equilibria, one of which is the steady-growth one and the other of which is the currency-crisis one accompanied by balance-sheets crisis as Krugman suggested. I examine conditions for the existence of self-fulfilling crisis equilibrium and find that an economy is likely to go into such crises if (1) propensity to import is low, (2) propensity to consume is low, (3) world interest rate is low, (4) borrowing constraint of private sector is moderate, (5) financial market is restrictive and there is high entry barrier, (6) price elasticity of export is low, and (7) wage elasticity of labor supply is low.

I also consider the situation that entrepreneurs owe unexpected excess liabilities, and show how these excess liabilities cause a large but temporal depreciation of exchange rate inevitably in the model. The unexpected excess debt burdens often come from unexpected business slowdown and/or bubble burst, so that the model suggests that the unfounded euphoria is one of the causes of crises.

Tightening monetary policy aiming to avoid a sharp depreciation of nominal exchange rate causes a deflation and makes the balance-sheet problem worse. Expansionary fiscal policy can avoid the crisis only if the government raises a sufficient fund from abroad, but the required amount of fund may be extraordinary large if the borrowing constraint is moderate.

The rest of the paper is as follows: The next section builds the model. Section 3

characterizes the equilibrium. Section 4 examines the characteristics of the economy which tends to be threatened by the fear of the crisis. Section 5 argues policy implications. A final section concludes with some remarks.

2. The Model

The model is a modified version of Krugman (1999). Time is discrete. There is a continuum of workers whose time horizon is only one period. The population is normalized to unity. There is also a continuum of entrepreneurs whose time horizon is forever. The population is also normalized to unity. There are two different kinds of goods, domestic goods and import goods. Domestic goods are supplied competitively by domestic firms. Workers consume both domestic and import goods. Entrepreneurs consume only import goods, but they use domestic goods to produce capital. They are under the borrowing constraint since they can promise to repay only up to a fraction of their future earning. There is no uncertainty and we assume agents have perfect foresight about future, except the relative price of import goods (= the real exchange rate) at the initial period.

2.1. Production technology of domestic goods

Firms are homogeneous and perfectly competitive, and maximize their profit for both wage w_t and capital rental price R_t taking as given. The production function of the representative firm is

$$y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{1}$$

where y_t is output, K_t is capital input and L_t is capital input. For simplicity, I assume that the level of productivity A_t is positively related to the average capital-labor ratio $k_t \equiv K_t/L_t$,

 $A_t = Ak_t^{1-\alpha},$

so that capital rental price R_t are constant over time:

$$w_t = (1 - \alpha)A_t K_t^{\alpha} L_t^{1-\alpha} = (1 - \alpha)Ak_t, \qquad R_t = \alpha A_t K_t^{\alpha - 1} L_t^{1-\alpha} = \alpha A.$$
(2)
2.2. Workers

Homogeneous workers' utility function is

$$u_{wt} = c_{dt}^{1-m} c_{it}^{m},$$

where u_{wt} is the level of utility, c_{dt} is consumption of domestic goods and c_{it} is that of import goods. Each worker supplies one unit of labor inelastically, so that his budget constraint is

$$c_{dt} + e_t c_{it} = w_t,$$

where e_t is the price of a import good relative to a domestic good. e_t means the real exchange rate. The solution of utility maximization problem is

$$c_{dt} = (1-m)w_t, \qquad c_{it} = \frac{mw_t}{e_t}.$$
(3)

2.3. Entrepreneurs

Homogeneous entrepreneurs consume only import goods, and their period utility function is

$$u_{et} = \ln c_{et},$$

where u_{et} is the level of utility and c_{et} is consumption of import goods. The representative entrepreneur maximizes the discounted sum of current and future utilities $\{u_{et}\}_{t=1}^{\infty}$, $\sum_{t=1}^{\infty} \beta^{t-1} u_{et}$, for given k_1 , b_1 and $\{e_t\}_{t=1}^{\infty}$, subject to the following three constraints:

$$k_{t+1} \ge (1 - \delta)k_t \tag{4}$$

$$e_{t}c_{et} + k_{t+1} - (1 - \delta)k_{t} = \begin{cases} a_{t} + e_{t}b_{t+1} - (1 - \delta)k_{t+1} & \text{if } a_{t} \ge 0\\ 0 & a_{t} < 0 \end{cases}$$
(5)

$$(1+r^{*})e_{t+1}b_{t+1} \le \rho(1+R_{t+1}-\delta)k_{t+1} \qquad \text{if} \quad a_t \ge 0 \tag{6}$$

$$(1+r)e_{t+1}b_{t+1} = \rho(1-\delta)(1+R_{t+1}-\delta)k_t \qquad \text{if} \quad a_t < 0 \tag{7}$$

where β denotes the discount factor, r^* is world real interest rate, δ is the depreciation rate of capital, a_t is the net worth, given by

$$a_t = (1 + R_t - \delta)k_t - (1 + r^*)e_t b_t, \tag{8}$$

 $k_t (\geq (1 - \delta)k_{t-1})$ is the amount of capital and $b_t \ge 0$ is the amount of debt at the beginning of period *t*. Eq. (4) means that the entrepreneur cannot sell any amount of her capital. Only the initial owner can utilize her capital. Eq. (5) is the budget constraint. Eq. (6) is the borrowing constraint when she has positive net worth.¹ Some of their income is unobservable and they always hide it from their creditors, so that they can borrow at most as much as they can commit to repay.² Eq. (7) is the borrowing constraint when they have liabilities in excess of assets and go bankrupt. If they go bankrupt, they cannot consume or get any new loan and owe the debt just as much as they can commit to repay by the income from existing capital $(1 - \delta)k_t$. The constraints (5) and (7) are derived from the assumption that the financial collapse occurs if there is unexpected large depreciation of real exchange rate and all entrepreneurs go bankrupt.³ This is a crude assumption, but it is crucial for the existence of crisis equilibrium. Entrepreneurs go bankrupt if the current real exchange rate e_t becomes high enough.

¹ This is a kind of ex post collateral constraints. In the original model of Krugman (1999), the borrowing constraint is assumed to be an ex ante collateral constraint, which related to the current net worth of the debtor such that $e_tb_{t+1} \le \rho\{(1 + R_t - \delta)k_t - (1 + r^*)e_tb_t - e_tc_{et}\}$. It makes, however, the dynamics of the equilibrium path more complicated. It is true that the ex ante collateral constraints are hard to be derived from an explicit economic model with rational agents, though they are widely observed as margin clauses. See Mendoza and Smith (2006) p. 88.

^{88.} ² If both e_{t+1} and R_{t+1} are constant over time, Eq. (6) can be derived from a simple incomplete contract model. Details are shown in appendix.

³ This is a kind of ex ante collateral constraints.

Solving the maximization problem, it is easily found that the borrowing constraint Eq. (6) is binding if

$$(1+r^{*})g_{et+1} < 1+R_{t+1}-\delta, \tag{9}$$

where $g_{et+1} \equiv e_{t+1}/e_t$ is the gross depreciation rate of real exchange rate. It is also true that $e_t c_{et}$ becomes

$$e_t c_{et} = (1 - \beta)a_t$$

if

$$(1+r^*)g_{et+1} > \rho(1+R_{t+1}-\delta).$$
(10)

for all *t*. I assume that the following inequalities hold:

$$1 + r^* < \frac{1}{\beta} < 1 + \alpha A - \delta < \frac{1 + r^*}{\rho},$$
(11)

so that k_{t+1} becomes

$$k_{t+1} = \begin{cases} \frac{\beta(1+r^{*})g_{et+1}a_{t}}{(1+r^{*})g_{et+1} - \rho(1+R_{t+1} - \delta)} & \text{if} & a_{t} \ge 0\\ (1-\delta)k_{t} & a_{t} < 0 \end{cases}$$
(12)

If the initial net worth a_1 is positive, the real exchange rate e_t is constant and $R_{t+1} = \alpha A$ over time, then the sequence of $\{k_{t+1}\}_{t=1}^{\infty}$ becomes

$$k_2 = \frac{(1+r^*)a_1}{1+r^* - \rho(1+\alpha A - \delta)}$$
(13)

$$k_{t+1} = \frac{\beta(1-\rho)(1+r^{*})(1+\alpha A-\delta)k_{t}}{1+r^{*}-\rho(1+\alpha A-\delta)}$$
(14)

for all $t \ge 2$, and therefore the gross investment i_t , defined by

$$i_t \equiv k_{t+1} - (1 - \delta)k_t$$

becomes

$$i_{t} = \frac{\{\beta(1-\rho)(1+r^{*})(1+\alpha A-\delta) - (1-\delta)\{1+r^{*}-\rho(1+\alpha A-\delta)\}k_{t}}{1+r^{*}-\rho(1+\alpha A-\delta)} > 0.$$

2.4. Export

For simplicity, I assume that export x_t is proportional to the output y_t such that

$$x_t = x^* e_t y_t, \tag{15}$$

where x^* is the parameter about the degree of foreign demand for domestic products. In order to set the steady-state value of exchange rate equal to one, x^* is set equal to the degree of dependence on export at the steady state:

$$x^{*} \equiv \alpha + m(1 - \alpha) + \frac{1 - \delta}{A} - \frac{\beta(1 - \rho)(1 + r^{*})(1 + \alpha A - \delta)}{A\{1 + r^{*} - \rho(1 + \alpha A - \delta)\}}$$
(16)

3. Equilibrium

The remaining condition for equilibrium is market clearing for domestic goods:

$$y_t = c_{dt} + i_t + x_t.$$
 (17)

Substituting Eqs. (1), (2), (3), (12) and (15) into (17) and rearranging, we obtain

$$e_{t} = \frac{1}{x^{*}} \left[\alpha + m(1 - \alpha) + \frac{1 - \delta}{A} - \frac{\beta(1 + r^{*})g_{et+1}a_{t}}{A\{(1 + r^{*})g_{et+1} - \rho(1 + \alpha A - \delta)\}k_{t}} \right]$$
(18)

for all $t \ge 2$. If the borrowing constraint (6) is binding on the perfect foresight equilibrium path, a_t must satisfy

$$a_{t} = (1 - \rho)(1 + \alpha A - \delta)k_{t} > (1 - \delta) \left\{ 1 - \frac{\rho(1 + R_{t+1} - \delta)e_{t}}{(1 + r^{*})e_{t+1}} \right\}k_{t}$$
(19)

from Eq. (9). We can find that, if b_1 is equal to $\rho(1 + \alpha A - \delta)k_1/(1 + r^*)$ so that a_1 is equal to $(1 - \rho)(1 + \alpha A - \delta)k_1$, there is a steady-growth equilibrium $\{k_t, e_t\}_{t=1}^{\infty}$ where k_{t+1} is given by Eq. (14) for all $t \ge 1$, and where e_t is given by

$$e_{t} = \frac{1}{x^{*}} \left[\alpha + m(1-\alpha) + \frac{1-\delta}{A} - \frac{\beta(1-\rho)(1+r^{*})(1+\alpha A-\delta)}{A\{1+r^{*}-\rho(1+\alpha A-\delta)\}} \right] = 1$$

for all $t \ge 1$, constant over time.

Next we consider the possibility of crisis equilibrium. Suppose that

$$b_1 = \frac{\rho(1+\alpha A-\delta)k_1}{1+r^*}$$

and $e_t = 1$ for all $t \ge 2$. Then the equilibrium value of (e_1, i_1) must satisfy both

$$i_{1} = \begin{cases} i(e_{1}, k_{1}) & e_{1} < \frac{1 + r^{*}}{\rho(1 + \alpha A - \delta)} (\equiv e_{B}) \\ \infty & \text{if} \quad \frac{1 + r^{*}}{\rho(1 + \alpha A - \delta)} \leq e_{1} \leq \frac{1}{\rho} (\equiv e_{L}) \\ 0 & e_{1} > \frac{1}{\rho} \end{cases}$$
(20)

and

$$e_{1} = \frac{1}{x^{*}} \left[\alpha + m(1 - \alpha) - \frac{i_{1}}{Ak_{1}} \right], \qquad (21)$$

where the function $i(e_1, k_1)$ is given by

$$i(e_1, k_1) = \left[\frac{1 + r^* - \rho(1 + \alpha A - \delta) \left\{\frac{\beta(1 + r^*) - (1 - \delta)}{\beta(1 + \alpha A - \delta) - (1 - \delta)}\right\} e_1}{1 + r^* - \rho(1 + \alpha A - \delta) e_1}\right] k_1.$$

IS curve is the graph of Eq. (20) and BP curve is that of Eq. (21) in Fig. 1. There is a crisis equilibrium $(e_1, i_1) = (e_c, 0)$ if and only if

$$e_c \equiv \frac{\alpha + m(1 - \alpha)}{x^*} > \frac{1}{\rho} \ (\equiv e_L) \tag{22}$$

holds. A large depreciation of real exchange rate will make financial collapse, which leads to sharp decrease of investment, and the decrease of investment must depreciate the real exchange rate.

4. Characteristics of the Economy with Crisis Equilibrium.

In this section, we examine what economy tends to suffer the Krugman's type of balance sheet crisis. We first examine how propensity to import, borrowing constraint and productivity affect the possibility of crisis. Then we make a close examination of the borrowing constraint in default. Finally the model will be extended so that we can examine how the price elasticity of export and the wage elasticity of labor supply affect the possibility of crisis.

4.1. Propensity to import, financial condition and productivity.

Eq. (22) must be satisfied if there is the crisis equilibrium. $\alpha + m(1 - \alpha)$ is the ratio of domestic goods consumption to output and x^* is the export-output ratio, so that Eq. (22) implies that the economy tends to come to the crisis if its dependence of exports is small. What economy depends on export little? We first examine how change in parameters affects Eq. (22). Define Δe by

$$\Delta e = \frac{\alpha + m(1 - \alpha)}{x^*} - \frac{1}{\rho}.$$
(23)

First we consider a small increase of workers' propensity to import m. Substituting Eq. (16) into (23), we obtain

$$\frac{\partial \Delta e}{\partial m} = \frac{\partial e_c}{\partial m}$$
$$= \frac{\partial}{\partial m} \left[1 - A^{-1} \{ \alpha + m(1 - \alpha) \}^{-1} \left\{ \frac{\beta (1 - \rho)(1 + r^*)(1 + \alpha A - \delta)}{1 + r^* - \rho (1 + \alpha A - \delta)} - (1 - \delta) \right\} \right]^{-1} < 0$$

since we assume Eq. (11). The crisis equilibrium tends to exist in the economy with smaller propensity to import. Small m means high propensity to consume domestic products and therefore little dependence on export.

Next we consider a small increase of the discount factor β . We can easily find that

$$\frac{\partial \Delta e}{\partial \beta} = - \frac{\alpha + m(1-\alpha)}{x^{*2}} \cdot \frac{\partial x^{*}}{\partial \beta} = \frac{\alpha + m(1-\alpha)\partial q_{i}}{x^{*2}} \cdot \frac{\partial q_{i}}{\partial \beta} < 0$$

where q_i is the ratio of investment to output on the steady-growth path and given by

$$q_{i} \equiv \frac{\beta(1-\rho)(1+r^{*})(1+\alpha A-\delta)}{A\{1+r^{*}-\rho(1+\alpha A-\delta)\}} - \frac{1-\delta}{A}.$$

 β is also the marginal propensity for gross savings, so that larger β means larger savings, which leads to larger investment. The crisis equilibrium tends to exist when the ratio of investment to output q_i is larger.

Next we consider a small increase of world interest rate r^* . We can easily find that

$$\frac{\partial \Delta e}{\partial r^*} = - \frac{\alpha + m(1-\alpha)}{x^{*2}} \frac{\partial x^*}{\partial r^*} = \frac{\alpha + m(1-\alpha)\partial q_i}{x^{*2}} < 0.$$

The world interest rate is the entrepreneur's borrowing rate and the lower interest rate makes investment larger. Therefore the depreciation of exchange rate at the crisis is larger if the world interest rate is lower and the entrepreneurs' outstanding debts are more.

Next we consider a small increase of parameter ρ which means the borrowing constraint loosens. We can easily find that

$$\frac{\partial \Delta e}{\partial \rho} = - \frac{\alpha + m(1-\alpha)}{x^{*2}} \cdot \frac{\partial x^{*}}{\partial \rho} = \frac{\alpha + m(1-\alpha)}{x^{*2}} \cdot \frac{\partial q_i}{\partial \rho} > 0,$$

which indicates that the crisis equilibrium does not exist in the economy with severe borrowing constraint. No crisis happens in an economy with too little investment.

The effect of small increase in productivity A, $\partial \Delta e / \partial A$, is ambiguous because its effect on the steady-state investment-output ratio q_i ,

$$\frac{\partial q_i}{\partial A} = \frac{\beta(1-\rho)(1+r^*)\{\rho(1+\alpha A-\delta)^2-(1-\delta)(1+r^*)\}}{A^2\{1+r^*-\rho(1+\alpha A-\delta)\}^2} + \frac{1-\delta}{A^2},$$

is ambiguous. $\partial q_i / \partial A$ is positive if $\rho(1 + \alpha A - \delta)^2 \ge (1 - \delta)(1 + r^*)$. It is plausible that productivity improvement increases investment and lowers the dependence on export. Thus we may say that the economy with high productivity is faced with the crisis.

The effect of small change in capital income share α is also ambiguous. The intuition is as follows: a small increase of α decreases the income share of workers and their domestic consumption, which raises the dependence on exports. The increase of α , however, raises the return on capital and investment. These two are the opposite effects on e_c , which is the lowest value of exchange rate consistent to crisis equilibrium.

4.2. A close examination of the assumption of financial collapse in Section 2.3.

There is one restrictive assumption in the model presented in Section 2.3. Eqs. (4) ~ (7) mean that no investment is made in the economy if all incumbent entrepreneurs go bankrupt at the same time ($a_t < 0$). This assumption includes the implicit assumption that no one can engage in any investment activity except the incumbent entrepreneurs. If the (expected) depreciation rate of exchange rate g_{et+1} is high enough to satisfy

$$g_{et+1} > \frac{\rho(1+R_{t+1}-\delta)}{1+r^*} \equiv g_{\min},$$

no entrepreneur without any net worth can invest at all since he cannot secure the required repayment. If, however, g_{et+1} is at most as much as g_{\min} , then entrepreneurs without any net worth can commit to repay all of their liabilities so that they can make investment in principle. Eq. (7) imposes an additional

constraint that defaulted entrepreneurs can borrow only for refunding the existing capital and get no new fund even if they have very profitable investment opportunities.

If there are some potential entrepreneurs (new comers) with no net worth, the crisis equilibrium does not exist. The reason is as follows: since the new comers are not in default of any debt, they can borrow as much as they want if they can repay. In the case of $g_{et+1} \leq g_{min}$, the return on capital denominated in foreign goods becomes very high so that the borrowing constraint of Eq. (6) is no longer restrictive and they can commit to repay any amount of loan without any net worth. Since the net profit from investment is strictly positive, the demand for loan becomes infinite if $g_{et+1} \leq g_{min}$. Therefore g_{et+1} must be more than g_{min} , which leads to $e_t < 1/g_{min}$ under the rational expectation of $e_{t+1} = 1$. Investment of the incumbent is, however, increasing in e_t until it reaches g_{min} . IS curve has no longer the vertical part and there is only one normal (non-crisis) equilibrium.

If the entrepreneurs in the model are considered as non-financial firms, Eqs. (4) \sim (7) are inappropriate assumptions since there are many potential new comers. On the other hand, if we interpret them as regulated financial institutions such as banks, an immediate entry into banking business is very difficult so that the assumptions Eqs. (4) \sim (7) may be persuasive. We can say that the economy with regulated and underdeveloped financial system, which heavily depends on small number of banks and implicit contracts, tends to be faced with the balance-sheet crisis.

4.3. Imported Investment Goods.

Entrepreneurs are assumed to use domestic product for their investment in Section 2.3. If they use imported one instead, then Eq. (17) must be replaced by and

 $y_t = c_{dt} + x_t, \tag{17'}$

so that, with replacing x^* by $x^* \equiv \alpha + m(1 - \alpha)$, the equilibrium exchange rate e_t is independent of the amount of investment and becomes unity for all *t*. This implies that Krugman's type of crisis is hard to occur in the economy where most of investment uses imported goods.

4.4. Price elasticity of foreign demand for domestic product.

In Section 2.4, the price elasticity of foreign demand for exports is modeled to unity. If we replace Eq. (15) by

$$x_t = x^* e_t^{\gamma} y_t, \tag{15'}$$

then Eq. (22) must be changed with

$$(e_c \equiv) \left\{ \frac{\alpha + m(1-\alpha)}{x^*} \right\}^{\frac{1}{\gamma}} > \frac{1}{\rho} \ (\equiv e_L).$$

$$(22')$$

The left-hand side of Eq. (22') is decreasing in the elasticity of foreign demand γ ,

so that the crisis tends to hit the economy where foreign demand for domestic products is price-inelastic.

4.5. Wage elasticity of labor supply

Labor supply was assumed to be supplied inelastically in Section 2.2. It may depend on the real exchange rate if it is elastic to real wage. As long as we assume that foreign demand for exports is in strict proportion to the actual output $Ak_t l_t^{1-\alpha}$, nothing changes by elastic labor supply. If we assume not only that labor supply l_t is given by

$$l_t = e_t^{-\delta m},\tag{24}$$

where δ is wage elasticity of labor supply, but also that export x_t is in proportion to the 'normal' output Ak_t at $l_t = 1$, then Eq. (22) becomes

$$(e_c \equiv) \left\{ \frac{\alpha + m(1-\alpha)}{x} \right\}^{\frac{1}{1+\delta m(1-\alpha)}} > \frac{1}{\rho} \ (\equiv e_L).$$

$$(22'')$$

The left-hand side of Eq. (22") is decreasing in the elasticity of labor supply δ , so that the economy with elastic labor supply (output) can avoid the crisis.

5. Economy with Excess Debt Burdens

In the previous section, we assume that the initial debt burdens b_1 is just as much as the representative entrepreneur can commit to repay in the normal equilibrium. In this section, we consider the entrepreneur with excess debt burdens, $b_1 > \rho(1 + R_1 - \delta)k_1$, because of unexpected changes in circumstance, such as productivity slowdown and bubble burst.

Let q_{b1} denote the ratio of initial debt to capital $(q_{b1} \equiv b_1/k_1)$. Then the equilibrium value of i_1 becomes

$$i_{1} = i(e_{1}, k_{1}, q_{b1}) = \left[\frac{1 + r^{*} - \left\{\frac{\beta(1 + r^{*})^{2}q_{b1} - \rho(1 - \delta)(1 + \alpha A - \delta)}{\beta(1 + \alpha A - \delta) - (1 - \delta)}\right\}e_{1}}{1 + r^{*} - \rho(1 + \alpha A - \delta)e_{1}}\right]k_{1} \quad (25)$$

as long as $e_1 < (1 + r^*)/\{\rho(1 + \alpha A - \delta)\}$. The left-hand side of Eq. (25) is strictly decreasing in e_1 if and only if

$$q_{b1} > \frac{\rho(1 + \alpha A - \delta)^2}{(1 + r^*)^2}.$$
 (26)

If Eq. (26) holds so that entrepreneurs have $100(\alpha A - \delta - r^*)/(1 + r^*)$ percent of excess liabilities, IS curve becomes a continues down-sloping curve as seen in Fig 2. In this case, there is a unique equilibrium with higher e_1 than the steady-state value. Caballero and Krishnamurthy (2006) shows how bubble-crashes cause capital flow reversals in an economy with underdeveloped financial market. Though there is no room for rational bubbles in the model of this paper, the model

suggests that the unfounded euphoria such as bubbles is one of the causes of crises since the problem of excess liability is often accompanied with unexpected business slowdown and/or bubble bursts.

6. Policy Implications

As we see in the previous sections, Krugman's model of currency and financial crisis is entirely a real one. Different from the first- and second-generation models of currency crisis, monetary policy is not a cause or medium of crisis in the model. It, however, has a potential ability to prevent a critical depreciation of nominal exchange rate by taking thorough deflationary measures. What happens with such policies in the crisis?

Since nominal exchange rate is unchanged, nominal burdens of debts denominated by foreign currency are also unchanged. Deflationary policy, however, makes nominal prices of domestic products lower, so that nominal sales of firms decrease. That is, deflationary policy causes a financial crisis through debt-deflation instead of currency crisis. It is a matter of course since Krugman's type of crisis is not a fall of the price of domestic currency, but a fall of relative price of domestic products. In order to raise the relative price of domestic products, it needs to increase demand for domestic product instead of aggregate demand.

Expansionary fiscal policy can be done only if the government can raises its fund from either workers or foreign investors or both in the model. There is no other source. Both taxation and bond issue to from workers are limited, so that it may be insufficient to avoid the crisis. It is also inefficient because they crowd out consumption of domestic product partly. If the government is ready to raise sufficient fund from foreign investors for its expenditure enough to lower the real exchange rate less than e_L at the crisis, it can eliminate the crisis equilibrium. The required amount of fund is, however, is very large nearly equal to the amount of lost investment if e_L is close to one, which is the exchange rate in the steady state equilibrium. Since e_L is decreasing in ρ , we may think that the crisis tends to hit the economy with partly developed financial market.

7. Concluding Remarks

This paper construct a simplified but complete version of Krugman (1999) model and examine what economy is likely go into crises. The model in this paper is completely non-monetary one, and the conclusion may be upset the result if the model extends to the monetary one. In this sense, this paper is still incomplete.

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Appendix: Derivation of Eq. (5)

Suppose that the entrepreneur can make sure to repay only a fraction of his income from capital because he can hide the rest and he cannot commit himself not to do. ρ_I denotes the secured repayment rate. Since the lender can hold a mortgage on the entrepreneur's capital, she can be repaid as much as he can borrow on the security of the capital at next period. Let $b_{t+1}(k_{t+1})$ denote the maximum amount of foreign currency that the entrepreneur with k_{t+1} amount of capital can borrow, and then $b_{t+1}(k_{t+1})$ satisfies

$$b_{t+1}(k_{t+1}) = \frac{\rho_l R_{t+1} k_{t+1} + e_{t+1} b_{t+2}((1-\delta)k_{t+1})}{(1+r^*)e_{t+1}}.$$
 (a1)

If both e_{t+1} and R_{t+1} are constant over time ($e_{t+1} = e$ and $R_{t+1} = R$ for all t), the function $b_{t+1}(\cdot)$ becomes time irrelevant and can be solved

$$b(k_{t+1}) = \frac{\rho_{l}Rk_{t+1}}{(1+r^{*})e} + \frac{b((1-\delta)k_{t+1})}{1+r^{*}}$$
$$= \frac{\rho_{l}Rk_{t+1}}{(1+r^{*})e} + \frac{(1-\delta)\rho_{l}Rk_{t+1}}{(1+r^{*})^{2}e} + \frac{(1-\delta)^{2}\rho_{l}Rk_{t+1}}{(1+r^{*})^{3}e} + \dots$$

$$= \frac{\rho_I R k_{t+1}}{(\delta + r^*)e}.$$

Let ρ define by

$$\rho \equiv \frac{\rho_I(1+r^*)R}{(\delta+r^*)(1+R-\delta)},$$

then we can obtain Eq. (5) with $e_{t+1} = e$ and $R_{t+1} = R$.





Fig. 2. Equilibrium with excess liabilities

