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INCOME TAXATION, MIGRATION, AND WORK INCENTIVES IN A DUAL ECONOMY MODEL

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I. INTRODUCTION

THIS paper analyzes the general equilibrium consequences of income taxation (both rural and urban) on internal migration and work incentives in a dual economy model. In spite of general agreement in the literature on the reasonableness of dual economy models as descriptions of developing economies, there exists hardly any analysis of the role of taxation (or, fiscal policy in general) in such a context.[1] This we believe is a glaring gap, and the present effort is intended to contribute to a better understanding of the role of government policies in the process of economic development.[2]

Migration, one of the two central issues focussed on in this paper, has been at the heart of most recent theoretical analyses.[3] The question raised here—How does income taxation in each sector influence rural-urban migration?—may be viewed as simply extending the discussion.

Our second concern is with the work-leisure choice. Here, we note that most dual economy models simply do not allow for this choice. In light of the seminal

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1 Authors such as Kelley, Williamson, and Cheetham [12] and Zarembka [23] have demonstrated that the predictions of their dual economy models are consistent with broad historical observations characterizing economic development. See Dixit [4] for a discussion of alternative models.

2 Contributions such as those contained in Bird and Oldman [3] are typically based on partial equilibrium frameworks and/or on behavioral hypotheses that are not derived from an explicit analytical framework. Hence they are not directly comparable to those contained herein. More on this below.

3 In addition to the contributions cited in footnote 1, a fairly comprehensive survey and additional references are to be found in Todaro [19].

contribution by Sen [17], the distinction between the number of workers and the number of work hours is an extremely important one. In particular, Sen showed that the possibilities of *surplus labor* (namely, the phenomenon of a reduction of the number of peasants *not* followed by a reduction of farm output) can best be understood by analyzing the work-leisure behavior of households. Hence a model that examines migration is severely incomplete if it does not incorporate the work-leisure choice since the consequences of migration on farm output critically depend on whether remaining family members offer additional work hours or not. Our interest in the role of taxation with respect to work-leisure choice is without precedence in the literature. The conventional analysis of income taxation and work-effort cannot be taken to shed much light on the question posed here. The former is partial equilibrium in nature and, further, is not meant to capture the rural-urban dualism.

One other aspect of the dual economy model presented here deserves attention. It has traditionally been assumed that the modern sector is a prototype industrialized economy which maximizes profits. We offer an alternative hypothesis, namely, worker management. Though not common in developing countries (except possibly in modern plantations), worker management has a long history in Europe. Many writers, notably Vanek [20], have predicted increased productivity under worker management and sharing of surplus. Recent contributions (e.g., [7]) support the Vanek position based on empirical research for Britain, France, Germany, and Italy. We analyze such a model primarily as an alternative hypothesis to profit maximization. We do, however, provide a comparison of our results to those under profit maximization assumptions.

Given reasonable hypotheses on the parameters of individual preferences and production technology,[4] the general equilibrium consequences of income taxation are straightforward. We find that increased rural (urban) income taxation leads to increased rural-urban (urban-rural) migration flows. Urban work incentive is also discouraged by the rural income tax. The rural work incentive of peasants is also discouraged under the additional assumption of Cobb-Douglas technology in that sector whether or not there is surplus labor. Urban income taxation likewise leads to a decreased supply of effort by peasants. All these results are *qualitatively* the same under *both* the worker-management and profit-maximizing versions of the dual economy models. The results differ between the two models when it comes to the effects on urban work incentive of increased urban income taxation. In the profit-maximizing version, incentive is discouraged. The same conclusion applies to the worker management model under the additional hypothesis of Cobb-Douglas technology in the urban sector.

The plan of the paper is as follows. In Section II, we present the basic dual economy model and explain the various hypotheses on preferences and technologies and relate them to the broad historical evidence on dualistic development. We discuss both the worker management and the profit-maximizing versions

[4] These hypotheses, to be explained in Section II of the paper, are based in part on empirical data and the historically observed pattern of dualistic development.

of the model. As a precondition of the general equilibrium analysis, in Section III, we explore the sectoral effects of taxation on the supply of effort by urban workers and peasants. This may also be viewed as of interest in its own right by way of complementing the existing literature on taxation and work-leisure choice. The general equilibrium effects of taxation on migration and the supply of effort (here accounting for both the direct and the indirect effects of migration) are analyzed in Section IV. Evaluation of the main results, in light of related studies both theoretical and empirical, and their policy relevance is taken up in the final section. Most of the derivations and other technical remarks are collected in the form of an appendix.

II. A LABOR-MANAGED DUAL ECONOMY

A. *Assumptions*

ASSUMPTION 1. The dual economy is open. One simplifying aspect of this assumption is that it enables us to choose the units such that the price of output in both sectors is (constant at) unity. On the appropriateness of such an assumption, see Stiglitz [18, p. 205].

ASSUMPTION 2. The modern sector is labor-managed. To keep things simple, we follow Vanek [20], Ward [22], and Domar [5] in assuming that firms are interested in maximizing the rate of dividend per worker. The rate of dividend, Y_u, is defined as[5]

$$Y_u = \frac{F(H_u) - R}{M}, \tag{1}$$

where $H_u = M(1 - l_u)$, and M denotes the number of identical workers in the modern sector, while l_u is the utility-maximizing level of leisure chosen by each worker; R is fixed rent paid by firms for capital services, say, to an outside agency (the state); and $F(H_u)$ is short-run production function in the modern sector with $F' > 0$, $F'' < 0$.

Notice that by allowing H_u to be optimally determined in the model (the optimal M being determined in general equilibrium through the migration process between the modern and the traditional sectors) and by not admitting any other variable factor in production, there is no real maximization involved on the part of the firm. It simply obeys the zero-profit condition stated by equation (1). In other words, equation (1) acts as a constraint in the worker's optimization problem.

ASSUMPTION 3. There are N identical peasants in the rural sector. The representative peasant solves the following optimization problem:

5 In our notation, the urban and rural sectors are denoted by subscripts and superscripts u and r. Whenever there is no possibility of confusion, they will be dropped.

$$\underset{\{Z_r,\, l_r\}}{\text{maximize}}\ U^r(Z_r, l_r), \tag{2}$$

$$\text{subject to } Z_r = \frac{(1-t_r)Q(H_r)}{N}, \tag{3}$$

where U^r is a quasi-concave utility function defined over net income Z_r, and leisure l_r, $U_1{}^r$, $U_2{}^r > 0$, $H_r \equiv (1-l_r)$ is the rural sector work effort, t_r is the rate of rural income taxation, and $Q(H_r)$ is the rural production function given a fixity of land[6] with $Q' > 0$, $Q'' < 0$.

ASSUMPTION 4. In the modern sector, the representative worker chooses leisure l_u by solving the following optimization problem:

$$\underset{\{Z_u,\, l_u\}}{\text{maximize}}\ U^u(Z_u, l_u), \tag{4}$$

$$\text{subject to } Z_u = (1-t_u)Y_u, \tag{5}$$

where Y_u = income per laborer as determined by equation (1),
t_u = rate of urban income taxation,
U^u = a quasi-concave utility function, $U_1{}^u$, $U_2{}^u > 0$.

The availability of work-leisure choice in the modern sector may be justified in light of empirical evidence that the urban sector suffers from a high rate of worker turnover and absenteeism [6]. This may, in part, be a reflection of occupational choice and hence, choice over hours of work and income.

ASSUMPTION 5. Rural work and urban work are mutually exclusive.

ASSUMPTION 6. The decision to migrate is undertaken on the basis of comparing the income-leisure trade-offs available in the two sectors in utility terms, that is, the cost of migration is zero.

ASSUMPTION 7. Total population is constant at $\bar{P}$, and is the sum of peasants and workers, i.e., $\bar{P} = M + N$.

Note that the above assumptions 5 and 6 do not preclude the possibility of reverse (urban to rural) migration, and indeed this may enhance the reasonableness of the work-leisure choice in the modern sector. The migration behavior generalizes the original Todaro hypothesis from expected income comparisons to one of utility levels. We feel that latter is more appropriate, given our objective of analyzing the effects of taxation on the structure of the dual economy. Finally, on the absence of unemployment (assumption 7), let us note that the above model is capable of generating "disguised unemployment," a phenomenon claimed by

6. From the form of equation (3) it is obvious that we are assuming that land is communally owned and thus there is no need to pay out any "rent" to anyone. There are other equivalent assumptions supporting our formulation (e.g., that land is equally divided among all peasants).

many to be common in LDCs. (Disguised unemployment may exist if withdrawal of a member from a peasant family induces those remaining to proportionately increase the hours per member to keep farm output constant. Here the marginal productivity per worker would appear to be nil.)

B. *Treatment of Tax Receipts*

It is assumed that the government uses the revenue collected to finance pure public goods (say, social overhead capital). Further, such benefits accrue to each agent (peasant, worker, or firm) in an additively separable manner. Hence it does not affect the decisions that these agents make at the margin.

C. *Equilibrium Conditions*

As a solution to the problem described by optimization problem (2) and (3), the peasant's equilibrium conditions are given by

$$\frac{U_2{}^r(Z_r, l_r)}{U_1{}^r(Z_r, l_r)}=(1-t_r)Q'(H_r) \qquad (6)$$

and by equation (3). Thus, in equilibrium, peasants equate the net marginal productivity of labor to the marginal rate of substitution between income and leisure.

In equilibrium, the urban workers equate

$$\frac{U_2{}^u(Z_u, l_u)}{U_1{}^u(Z_u, l_u)}=(1-t_u)F'(H_u). \qquad (7)$$

Equation (7) states that, in equilibrium, workers in the labor-managed urban sector equate the marginal rate of substitution between income and leisure to the net (of tax) marginal (value) productivity of urban labor. Notice that conditions (6) and (7) are, as would be expected, fully symmetric.

Under assumptions 5 and 6, the condition necessary for a migration equilibrium is

$$U^u(Z^*_u, l^*_u)=U^r(Z^*_r, l^*_r), \qquad (8)$$

where the starred values are the solutions to the worker's and peasant's optimization problems.

The allocation of total population is governed by

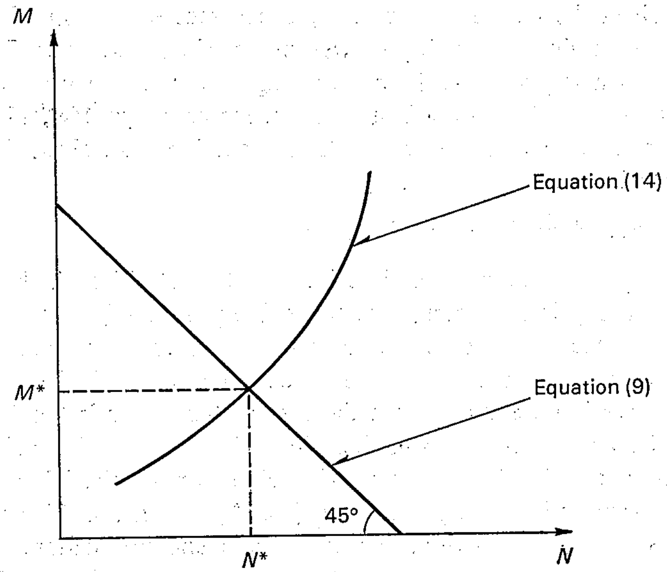
$$M+N=\bar{P}. \qquad (9)$$

D. *General Equilibrium*

The labor-managed dual economy can be fully described by the six equilibrium conditions stated above, namely, equations (3), (5), (6), (7), (8), and (9). The unknowns are $\{Z_r, l_r, Z_u, l_u, M,$ and $N\}$ and, in general, we are able to solve for them in terms of the parameters involved. However, we will often find it convenient to work with a "reduced form" of the above system. This is obtained as follows:

From equations (3) and (6), we obtain

Fig. 1. Equilibrium Allocation of Peasants and Workers



$$Z^*_r = Z_r(N, t_r) \tag{10}$$

and

$$l^*_r = l_r(N, t_r). \tag{11}$$

Likewise, from (5) and (7) we have

$$Z^*_u = Z_u(M, t_u) \tag{12}$$

and

$$l^*_u = l_u(M, t_u). \tag{13}$$

Substituting these starred values in equation (8) we have

$$V^u(M, t_u) = V^r(N, t_r), \tag{14}$$

where V^u and V^r are merely the indirect utilities by definition. Further, in view of equation (9), we can rewrite equation (14) as

$$V^u[M(N), t_u] = V^r(N, t_r). \tag{15}$$

The reduced form equation (15) summarizes the entire general equilibrium system.

Graphically, the general equilibrium can be shown rather easily (Figure 1). Equation (9) describes a negatively sloped 45° line in the (M, N) space. The other equilibrium condition (14), upon differentiation yields

$$\frac{dM}{dN} = V_N{}^r / V_M{}^u, \tag{16}$$

where the subscripts N and M denote the partial derivatives of indirect utilities, respectively, of changes in rural and urban population. From the discussion in Appendix C it is seen that both these derivatives are negative in our "general" model.[7] Clearly, the positively sloped dM/dN function given by equation (16) implies that, given the choice of tax rates, concurrent expansion of each sector is required to maintain the equality of well-being of each sector.[8] The constant level of population [equation (9)], in turn, suggests that as we move from left to right, the relative welfare of urban dwellers increases (as V_N^r, $V_M^u<0$). The intersection therefore gives us the population allocation that, given the tax rates, equates welfare in the two sectors.

E. *The "General" Model and Special Cases*

Before moving on to the next section, let us note that although the description of the dual economy is now complete, the subsequent analysis of taxation will prove more tractable, interpretable, and intuitive under some additional conditions on individual preferences and on production technologies. One set of conditions that we impose throughout the rest of the paper is the linear homogeneity of utility and production functions in both sectors.[9] The main import of this assumption is that it allows simpler expressions for the elasticities of substitution in both consumption and production (see Appendix B). These elasticities are denoted σ_0 and σ, respectively, in consumption and production. (The superscripts u and r, as usual, denote the two sectors, respectively.) Thus, from hereon, the model as described above, when reinforced by the hypothesis of linear homogeneity, will be referred to as the "general" model.

In the context of this general model we derive various predictions regarding the impact of tax variables on the structure of the dual economy, principally on the migration pattern and on sectoral work effort. To highlight the main conclusions, we make frequent references to a hypothesis widely believed to be descriptive of a less developed economy, namely,

$$0<\sigma<\sigma_0<\infty \tag{17}$$

and, in particular,

$$\sigma_0>1. \tag{18}$$

Zarembka [23] is among those supporting the above position. He argues that the above relationships, conditions (17) and (18) are "a close approximation to reality" [23, p. 18]. Extensive empirical evidence cited by both Zarembka [23] and Kelley, Williamson, and Cheetham [12] suggests that the elasticity of substitution in the modern production sector σ^u systematically increases as development takes place, and that this elasticity in rural production σ^r is greater than

7 Below we provide a characterization of what will be referred to as the "general" model throughout the paper.

8 The second derivative, d^2M/dN^2, is rather complex however. The graph in Figure 1 (convex downward), therefore, is arbitrary.

9 Such an assumption need not be viewed as restrictive. In theoretical modeling, even more radical assumptions are common. The book by Zarembka [23] for example, assumes constant elasticity of substitution in preferences and Cobb-Douglas technologies in urban production and constant elasticity of rural output with respect to labor, the variable input in the rural sector.

that in urban production. Estimates of σ^r are frequently put at unity or larger.[10] The above evidence, combined with the belief that the elasticity of substitution in consumption is higher than that in production, can be taken to imply condition (18).

Indeed, the classical characterization of surplus labor [17] occurs when $\sigma_0 \rightarrow \infty$. It may be noted that with $\sigma_0 \rightarrow \infty$, we would have a *constant* marginal rate of substitution between income and leisure, and hence the "real" cost of an additional man-hour is also constant. Thus when a member of a peasant family leaves, the remaining members proportionately increase the hours supplied with the consequence that the total hours supplied on the farm are constant, hence output remains unchanged. Although it is not a hypothesis that we maintain in this paper, we will interpret our results for this special case where this is of interest.

Another special case that we will comment on is where production in the modern or in the rural sector follows the Cobb-Douglas technology (σ^u, $\sigma^r = 1$). Apart from the algebraic simplicity gained by such a hypothesis, we may interpret this as follows: In light of the preceding discussion of the difference in the magnitude of elasticity between the two sectors, namely, that with economic development σ^u rises (perhaps from a number close to zero) and σ^r falls (from a number larger than unity), these two elasticities may very well approach unity. Kelley et al. remarks: "it appears plausible that production dualism. . . may diminish as development takes place" [12, p. 227]. Thus the Cobb-Douglas technology may be a relevant description at an advanced level of dualistic development.

Finally, we have the case of a profit-maximizing modern sector. Returning to the general model, the alternative hypothesis of profit maximization entails only a few changes. Equation (1) would now be replaced by

$$Y_u = wh_u, \tag{1'}$$

where w is the competitive urban wage. We thus add a new variable, the wage rate. To complete the model, we would also add the maximization condition, namely,

$$F'(H_u) = w. \tag{19}$$

After appropriate substitutions, the entire model can be represented by the following two conditions in place of equation (15):

$$V^u(w, t_u) = V^r(N, t_r) \tag{15'}$$

and

$$F'[M(N) \cdot h_u(w, t_u)] = w. \tag{19'}$$

In the subsequent analysis of taxation we shall present the results for both versions, namely, worker management and profit maximization. However, in order to economize on notations, we show the derivation only for the former.

10 See Griliches [9] and other references cited in Zarembka [23] and Kelley et al. [12].

III. LABOR SUPPLY IN PARTIAL EQUILIBRIUM

Before we undertake the general equilibrium analysis of income taxation on labor supply in each sector (both at the individual and collective level, the latter through migration), as an intermediate step we review the effects of taxation and migration on labor supply in partial equilibrium, i.e., at the sectoral level (i.e., each sector considered in isolation). Since these results also provide a detailed understanding of individual labor supply decisions in a developmental framework, they are of interest in their own right. One major difference between the present analysis and the standard work-leisure choice model is that work-leisure trade-off is non-linear here (due to production relations in the economy).[11]

A. *Work Incentives in the Rural Sector*

From the peasant's work-leisure choice problem [given by optimization problem (2) and (3) above], we obtain

$$\frac{\partial l_r}{\partial t_r}=\frac{l_r(1-l_r)(\sigma_0{}^r-1)}{(1-t_r)\{1+l_r[\gamma^r\sigma_0{}^r+(\alpha^r-1)]\}}, \tag{20}$$

$$\frac{\partial l_r}{\partial N}=\frac{l_r(1-l_r)\left(\dfrac{1}{\sigma^r}-\dfrac{1}{\sigma_0{}^r}\right)}{N\left[\dfrac{l_r}{\sigma^r}+\dfrac{1}{\sigma_0{}^r}\left(\dfrac{1}{1-\alpha^r}-l_r\right)\right]}. \tag{21}$$

(All derivations are discussed in Appendix B.) In addition to the notation already introduced, α is the elasticity of net output[12] with respect to total labor hours, i.e., $\alpha^r\equiv[H_rQ'(H_r)]/Q$, and γ is the elasticity of marginal productivity of labor, i.e., $\gamma^r\equiv(-)[H_rQ''(H_r)/Q'(H_r)]$.[13] It may be noted that due to linear homogeneity of the production function, $0<\alpha<1$ and, due to its concavity, $0<\gamma\leq 1$ in both sectors.

The interpretation of these results is straightforward. First, the effect of taxation on the supply of effort [equation (20)]. The general indeterminacy is due to the familiar conflict between the income and substitution effects induced by taxation. However, with the linearly homogeneous utility function, we do have a direct comparison between the two effects not so far discussed in the literature: *The supply of effort by a representative peasant decreases, remains the same, or increases due to increased taxation of rural income according as the elasticity of substitution between income and leisure is greater than, equal to, or less than,*

[11] Except in the "alternative" (profit-maximizing) model where the modern sector work-leisure trade-off is indeed linear [equation (1')].

[12] Note that we have defined $Q(H_r)$ to be the net rural output (gross output minus land rents). Similarly, for the urban sector, $(F-R)$ is the net output (gross output minus capital charges), and $\alpha^u\equiv[H_uF'(H_u)]/(F-R)$.

[13] Similarly, we would define $\gamma^u\equiv(-)[H_uF''(H_u)/F'(H_u)]$. It may be noted that under linear homogeneity, $\sigma=(1-\alpha)/\gamma$.

unity. Further, in the context of the hypothesis proposed in this paper [condition (18)], an increase in the tax rate would decrease the supply of effort per peasant. We thus have the intuitive result that with a "high" elasticity of substitution between income and leisure, the substitution effect dominates the income effect. Indeed, it may be seen that as $\sigma_0{}^r \rightarrow \infty$, the income effect tends to zero (the "surplus labor" condition), and we are left with the substitution effect alone, a result that was noted by Sen [17].

The usual conflict between income and substitution effects is again present in the effect of migration on the peasant's supply of effort. It can be seen that the substitution effect arises due to the increase in the return to effort, measured by Q' (since $Q''<0$), consequent upon a withdrawal of peasants. The substitution effect of migration (a fall in N) therefore is qualitatively the opposite of a rural income tax: it increases the supply of effort by the remaining members. The overall effect, however, in our general model, is: *A decrease in the number of peasants increases, leaves the same, or decreases the peasant's equilibrium work hours according as the elasticity of substitution in consumption (income-leisure) is greater than, equal to, or less than that in production (land-labor).* Given our hypothesis on these elasticities [condition (17)], we would expect the supply of effort to increase with increased rural to urban migration, again due to a dominant substitution effect. Elsewhere Ali [1] has shown that $\sigma_0{}^r \rightarrow \infty$ is a necessary and sufficient condition for work hours per member to increase *proportionately* with a decrease in family size.[14] In the latter event, total hours supplied and hence total farm output remain unchanged, which is the classical surplus labor characterization.

B. *Work Incentive in the Modern Sector*

From optimization problem (4) and (5), it is evident that the effects of urban income taxation and migration on the urban supply of effort will be equivalent to equations (20) and (21). Our version of worker management makes the choice problems in both the traditional and modern sectors symmetric. Thus we obtain

$$\frac{\partial l_u}{\partial t_u} = \frac{l_u(1-l_u)(\sigma_0{}^u - 1)}{(1-t_u)\{1+l_u[\gamma^u \sigma_0{}^u + (\alpha^u - 1)]\}} \tag{22}$$

and

$$\frac{\partial l_u}{\partial M} = \frac{l_u(1-l_u)\left(\frac{1}{\sigma^u} - \frac{1}{\sigma_0{}^u}\right)}{M\left[\frac{l_u}{\sigma^u} + \frac{1}{\sigma_0{}^u}\left(\frac{1}{1-\alpha^u} - l_u\right)\right]}. \tag{23}$$

The interpretation of these results can proceed in a manner identical to the above.

Let us also note that in the alternative model with a profit-maximizing modern

[14] In Sen's own model, this condition was sufficient to generate the surplus labor result. In the present context, it is *both* necessary and sufficient. Zarembka also derives this result in a model somewhat less general than ours. (He has CES production and utility functions.)

sector, the corresponding problem given by equation (1′) and optimization problem (4), the size of the sector M does not affect the work-leisure choice of a typical urban worker. The effect of increased urban income taxation on the supply of effort can, however, be investigated. This, in fact, is the standard treatment of work-leisure choice discussed at length in the literature since Robbins [15]. Given linear homogeneity of the utility function, it can be seen that, *qualitatively*, this result is the same as that given above by equation (22). In other words, for $\sigma_0^u>1$, increased urban income taxation discourages the supply of effort (as the substitution effect dominates the income effect).

To summarize this section, we note the main results. Given our hypothesis on the relative magnitude of the elasticities of substitution in consumption and production, we find that increased income taxation leads to a decrease in the supply of effort both by the peasant and the urban worker. The decrease is independent of whether the modern sector is of a worker-management or profit-maximizing variety. In a similar vein, we find that with increased rural-urban migration (i.e., as N falls and M increases) the supply of effort by the peasant increases and that by the urban worker decreases.

IV. GENERAL EQUILIBRIUM EFFECTS OF TAXATION

Here we analyze the effects of taxation on migration pattern and on work incentive (measured by the supply of effort) of workers and peasants in a general equilibrium context. It should be evident from the analysis so far that the results would be symmetric between the two sectors in the worker-management version of the dual economy. Thus we present detailed analysis for the rural income tax only in the text. Effects of the urban income tax are also stated but the derivation is to be found in Appendix D. The policy implications of the results of our analysis are explored in the final section.

A. *Rural Income Tax*

1. *Effect on migration*

First we analyze the effect of the rural income tax on migration. Not only is this of primary interest in itself, it also enables us to evaluate the consequences of the tax on the supply of effort. From equation (15), we have

$$\left(\frac{\partial V^u}{\partial M}\frac{\partial M}{\partial N}\right)\cdot\frac{dN}{dt_r}=\frac{\partial V^r}{\partial N}\cdot\frac{dN}{dt_r}+\frac{\partial V^r}{\partial t_r}$$

or

$$\frac{dN}{dt_r}=\left(\frac{V_t^r}{V_n^u-V_n^r}\right)<0. \tag{24}$$

The notation and sign of the partial derivatives of the indirect utility function are discussed in Appendix C. Increased taxation clearly lowers utility in a direct manner ($V_t^r<0$). The indirect effect through migration is in terms of returns to effort in production. With a decrease in N, marginal product rises, and hence per capita share and utility rises ($V_n^r<0$). Decreased N (implying increased M) would have the opposite effect in the urban sector ($V_n^u>0$). We thus have the following results:

PROPOSITION 1. The general equilibrium impact of an increase (decrease) in the rural income tax rates is to reduce (increase) the equilibrium number of peasants, i.e., increase (decrease) migration to the cities.

2. *Effect on urban work-effort*

Urban demand for leisure can be written as

$$l^*_u = l_u\{M[N(t_u, t_r)], t_u\}.$$

[Clearly, by equation (15), N must depend on *both* t_u and t_r]. Upon differentiation, however, we have[15]

$$\frac{dl^*_u}{dt_r} = (-)\frac{\partial l_u}{\partial M}\frac{dN}{dt_r}. \tag{25}$$

Given equation (24),

$$\text{sign } \frac{dl^*_u}{dt_r} = \text{sign } \frac{\partial l_u}{\partial M}.$$

Further, given equation (23), we can state

$$\frac{dl^*_u}{dt_r} \gtreqless 0 \quad \text{as} \quad \sigma_0{}^u \gtreqless \sigma^u. \tag{26}$$

In this model, rural income tax affects urban supply of effort only through its effect on migration. Since increased taxation leads to an expansion of the urban sector, urban work incentives are encouraged or discouraged depending on whether the income effect (inversely proportional to σ_0) dominates or is dominated by the substitution effect (inversely proportional to σ).[16] As noted earlier, the latter arises due to reduced return to effort due to an increase in M. In light of the proposed hypothesis on elasticities of substitution, we can state the above result as follows.

PROPOSITION 2. Increased rural income taxation decreases the general equilibrium supply of effort by urban workers.

The general result, however, is as stated by the relationship (26).

3. *Effect on rural work-effort*

Again, the peasant's equilibrium supply of effort may be written as

$$l^*_r = l_r[N(t_u, t_r), t_r],$$

which upon differentiation yields

15 As t_u and t_r are unrelated, we have the simpler form of this derivative.

16 Comparing equations (B.10) and (B.11) of Appendix B, the relationship of (26) and the relative magnitude of the income and substitution effects become transparent.

$$\frac{dl^*_r}{dt_r}=\frac{\partial l_r}{\partial N}\frac{dN}{dt_r}+\frac{\partial l_r}{\partial t_r}. \tag{27}$$

As would be expected, the total effect on the supply of effort consists of the direct (partial equilibrium) effect (second term on the right) *and* the indirect effect through changes in migration (the first term). The three expressions on the right are given by equations (21), (24), and (20), respectively. That analysis of equation (27) is complex is evident from a consideration of the hypothesis on elasticities advanced in this paper. In such a world, the direct partial equilibrium effect of increased taxation is to decrease the supply of effort (l_r goes up). Taxation also lowers the number of peasants, and this lowering of the number of peasants induces the remaining members to increase the supply of effort (l_r goes down). Thus the direct effect discourages and the indirect effect (via migration) encourages work incentive. Hence the ambiguity.

It may be further pointed out that the hypothesis advanced on the relative magnitude of the elasticities of substitution, in effect, allows us to directly compare income and substitution effects implicit in the derivatives ($\partial l_r/\partial t_r$ and $\partial l_r/\partial N$). We cannot thus expect any additional easily interpretable results at the general level. However, if we abandon our hypothesis on substitution elasticities, the total effect would be to discourage work incentives if $\sigma^r>\sigma_0{}^r>1$. (The first part of the inequality renders $\partial l_r/\partial N<0$, while the second part requires $\partial l_r/\partial t_r>0$.) Similarly, if this inequality were reversed, we would get the opposite result. The point we want to make is that conditions such as $\sigma^r>\sigma_0{}^r$ are counter-intuitive and hence not of interest in describing a developing economy.

There are, however, at least two special cases where the above result is considerably simplified. Both of these arise when we have Cobb-Douglas technology in the rural sector (implying $\sigma^r=1$). It may be noted that there is a growing body of evidence indicating that this elasticity is very close to unity [23, pp. 174–80]. We also have mentioned that even where $\sigma^r>1$, it is believed that with development, this is likely to fall [12]. In order to see how equation (27) applies to this case, we substitute equations (21), (24), and (20) into equation (27) and simplify to obtain

$$\frac{dl^*_r}{dt_r}=\frac{l_r(1-l_r)\left[(1-\alpha^r)(1-t_r)A^r\left(\frac{1}{\sigma^r}-\frac{1}{\sigma_0{}^r}\right)+N\left(1-\frac{1}{\sigma_0{}^r}\right)\right]}{N(1-\alpha^r)(1-t_r)B^r}, \tag{28}$$

where

$$A^r\equiv\frac{dN}{dt_r}<0,$$

$$B^r\equiv\left[\frac{l_r}{\sigma^r}+\frac{1}{\sigma_0{}^r}\left(\frac{1}{1-\alpha^r}-l_r\right)\right]>0.$$

Therefore, the sign of dl^*_r/dt_r depends on the sign of

$$(1-\alpha^r)(1-t_r)A^r\left(\frac{\sigma_0{}^r-\sigma^r}{\sigma^r}\right)+N(\sigma_0{}^r-1). \tag{29}$$

As $\sigma^r = 1$, (29) simplifies considerably. First, note that $(1-\alpha^r)(1-t_r)$ equals $N(V_n{}^r/V_t{}^r)$, using equations (C.1) and (C.3) of Appendix C. Thus (29) reduces to

$$N(\sigma_0{}^r - 1)\frac{V_n{}^u}{V_n{}^u - V_n{}^r} \tag{30}$$

and

$$\left.\frac{dl^*{}_r}{dt_r}\right|_{\sigma^r=1} \gtreqless 0, \quad \sigma_0{}^r \lesseqgtr 1. \tag{31}$$

Given our hypothesis that $\sigma_0{}^r > 1$, we would thus expect work incentive to be discouraged by taxation. Heuristically, this may be explained as follows. The Cobb-Douglas assumption imposes a quantitative restriction on the substitution effect generated by the decreased number of peasants. Consequently, the relative magnitudes of both income and direct and indirect substitution effects become comparable. The final conclusion suggests that net discouragement of incentive through the direct effect $(\partial l_r/\partial t_r)$ dominates the indirect encouragement effect through migration $(\partial l_r/\partial N)$.

The second special case obtains when we assume surplus labor in the rural sector along with the Cobb-Douglas hypothesis. From equation (28) it is obvious that if $\sigma^r = 1$ and $\sigma_0{}^r \to \infty$, the sign of $dl^*{}_r/dt_r$ depends on the sign of

$$\left(\frac{V_n{}^u}{V_n{}^u - V_n{}^r}\right) > 0 \Rightarrow \left.\frac{dl^*{}_r}{dt_r}\right|_{\substack{\sigma_0{}^r \to \infty \\ \sigma^r = 1}} > 0. \tag{32}$$

As noted earlier with $\sigma_0{}^r \to \infty$, the income effects vanish from both the direct and indirect effects of taxation. But with $\sigma^r = 1$, the remaining substitution effects are comparable and, again, the direct effect dominates.

The following proposition summarizes the main results on rural work incentives.

PROPOSITION 3 (Cobb-Douglas technology): Increased rural income taxation discourages equilibrium work incentive of peasants both (a) where $\sigma_0{}^r > 1$, and (b) where $\sigma_0{}^r \to \infty$ (surplus labor).

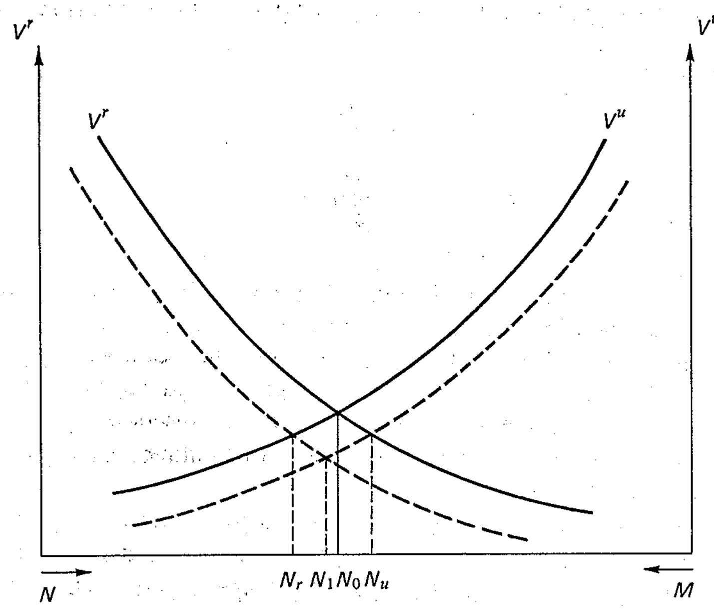
Thus given our hypotheses, Cobb-Douglas technology is sufficient to predict a decreased supply of effort by peasants whether or not surplus labor characterizes agriculture.

Before concluding the discussion of the rural income tax, let us note that in the alternative model with a profit-maximizing modern sector, all the results mentioned above, namely, those on migration and work incentive, remain *qualitatively* unchanged. This can be seen by differentiating equations (15′) and (19′) and proceeding as above.

B. *Urban Income Tax*

The analysis of the urban income tax is analogous to that of the rural income tax. A brief algebraic statement of the main results is contained in Appendix D.

Fig. 2. Taxation and Migration



Here we recapitulate the main conclusions, the interpretations of which proceed in the same manner as in the preceding section.

PROPOSITION 4 (Migration): Increased urban income taxation induces reverse migration (urban-rural).

PROPOSITION 5 (Urban work incentive): Increased urban income taxation discourages the equilibrium supply of effort by urban workers where modern sector production follows the Cobb-Douglas technology and our hypothesis on substitution elasticities applies (i.e., $\sigma_0{}^u > 1$).

PROPOSITION 6 (Rural work incentive): Increased urban income taxation discourages the rural supply of effort given our hypothesis on substitution elasticities ($\sigma_0{}^r > \sigma^r$).

Finally, let us note that in the alternative model with a profit-maximizing modern sector, propositions 4 and 6 remain intact. The result on urban work incentive simplifies greatly. We obtain the same result as stated above *without* the Cobb-Douglas assumption ($\sigma^u = 1$).

The effects of taxation on migration can also be illustrated by means of Figure 2. Given the directions of the partial derivatives, both V^r and V^u would be drawn as above. N_0 is the initial equilibrium at some given level of tax rates. As t_r

TABLE I

EFFECTS ON MIGRATION DECISIONS IN THE WORKER-MANAGEMENT AND PROFIT-MAXIMIZING VERSIONS OF THE DUAL ECONOMY MODEL

Type of Tax \ Effects on	Equilibrium No. of Workers	Equilibrium No. of Peasants
Urban income tax	−	+
Rural income tax	+	−

TABLE II

EFFECTS ON WORK INCENTIVES

Type of Tax \ Effects on	Equilibrium Work Hours per Worker	Equilibrium Work Hours per Peasant
Urban income tax	Cobb-Douglas production function in the modern sector (Only under worker management)* Decreases / Unchanged / Increases as $\sigma_0^u \gtreqless 1$	Decreases / Unchanged / Increases as $\sigma_0^r \gtreqless \sigma^r$
Rural income tax	Decreases / Unchanged / Increases as $\sigma_0^u \gtreqless \sigma^u$	(i) Cobb-Douglas production function in the rural sector Decreases / Unchanged / Increases as $\sigma_0^r \gtreqless 1$ (ii) Surplus labor plus (i) Decreases

* As noted in the text, for the urban income tax, the Cobb-Douglas assumption is required only in the worker-management model.

is increased, V^r falls (dashed curve). Similarly for t_u. Equilibrium allocations N_r and N_u, obtain when only one tax is adjusted. If both are raised simultaneously, the outcome N_1 could be on either side of N_0 depending on the relative magnitudes. Propositions 1 and 4 simply indicate the locations of N_r and N_u, respectively.

To sum up, then, we find that in our general model (in conjunction with the hypotheses on substitution elasticities) increased taxation of rural income induces rural-urban migration in both the worker-management and profit-maximizing versions of the dual economy model. The same tax discourages urban work incentive, again in both models. Rural work incentive is also discouraged (in both models) when we have Cobb-Douglas technology in the rural sector (whether or not there is surplus labor). Urban income tax, likewise, leads to reverse migration and a decreased supply of effort in both models. When it comes to rural work incentive, the effects of urban taxation differs between the two models. In the profit-maximizing version, we obtain a decreased supply of effort. In the worker-management model, the same conclusion follows only under the additional restriction that urban production technology be of the Cobb-Douglas variety. Tables I and II sum up these results in an exhaustive manner.

V. POLICY IMPLICATIONS

In this section we explore the policy implications of the main results arrived at in the preceding section. We do this by first relating our results to the existing literature, especially those embodying empirical verifications. Since our results fall in two categories, namely, migration and the supply of effort, we consider them in turn.

A. *Migration*

Clearly our analysis suggests that increased rural-income taxation leads to greater flows of rural-urban migration. The development literature on migration recognizes this eventuality [19, p. 3]. Since taxation directly affects rural and urban incomes (wages), the empirical literature on the responsiveness of migration flows to wage rates is of direct interest. Several studies, e.g., Greenwood [8] on India and Schultz [16] on Venezuela, report statistically significant data supportive of our results on migration.

Although taxation has other effects, insofar as migration behavior is concerned, it would appear reasonable to conclude that if urban job creation appears to support existing migration flows, tax treatment of urban and rural sectors should be adjusted to keep the relative real incomes unchanged. In other words, if the goal is to curb migration, urban workers may have to be taxed more heavily than are peasants. Thus taxation may be viewed as a tool available to policymakers to help regulate the migration behavior. At the same time the output and productivity consequences of taxation should not be ignored. This is the issue to which we now turn.

B. *Supply of Effort and Output*

The traditional belief concerning taxation of rural incomes (or, generally, taxation in developing countries), appears to be that the rural sector (landowners) is not taxed adequately. It is further believed that increased agricultural taxation (based on agricultural output, land values, or some combination of the two) would lead to greater efficiency in rural resource allocation (say, through more productive use of land, more intensive cultivation, and transfer of land from inefficient to efficient farmers). While Kaldor [10] [11] is most widely associated with the above views, writers such as Lewis [13] and Prest [14] have also adhered to the same reasoning. It should be noted that these beliefs are largely intuitive, not derived from a formal analysis. We interpret this view, namely, the positive productivity gain stemming from taxation, to be what we have labelled the indirect effect of taxation working *via* migration. In our analysis, this positive influence on productivity arises as a reduced number of peasants responds to higher return to their effort (as $Q''<0$). The thrust of our conclusion (for both the worker-managed and profit-maximizing models) is that the net effect on effort is negative (given our belief on the relative strengths of income and substitution effects). We also find that migration flows increase with rural taxation. The combined effect of both would be an overall reduction in the supply of effort in agriculture H_r and hence, reduced rural output. Thus our general conclusion appears to contradict the traditional belief.

It should, however, be pointed out that we have only analyzed income taxation. It is conceivable that alternative taxation policies (e.g., direct taxation of land) would lead to different results. A comparison of alternative taxes should form part of the agenda for future research. To conclude, then, we would recommend that until better empirical evidence becomes available on the various parameters involved in theoretical analysis, and pending exhaustive theoretical analysis of alternative taxation policies, the efficiency-generating possibilities of rural income taxation should be treated rather cautiously.

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APPENDIX A

LINEARLY HOMOGENEOUS PREFERENCES

Under the arguments for linear homogeneity of the utility function, we have, from Euler's theorem (see, for example, [2, p. 283]), the following conditions:[a]

$$\begin{aligned} &ZU_1+lU_2=U, \\ &ZU_{11}+lU_{12}=0, \\ &ZU_{21}+lU_{22}=0. \end{aligned} \tag{A.1}$$

The elasticity of substitution in consumption, between Z and l, σ_0, can be written in the following equivalent forms given linear homogeneity of preferences:

$$\sigma_0=(-)\frac{lU_1U_2}{ZUU_{11}}>0 \tag{A.2}$$

and

$$\sigma_0=(-)\frac{ZU_1U_2}{lUU_{22}}>0. \tag{A.3}$$

Further, we have, from equations (A.1), (A.2), and (A.3), two additional expressions for the inverse of the elasticity of substitution:

$$\text{(a)}\quad \frac{ZU_{12}}{U_2}-\frac{ZU_{11}}{U_1}=\frac{1}{\sigma_0} \tag{A.4}$$

and

$$\text{(b)}\quad \frac{lU_{21}}{U_1}-\frac{lU_{22}}{U_2}=\frac{1}{\sigma_0}. \tag{A.5}$$

[a] Since the results stated in this section are independent of sector, the superscripts have been omitted.

APPENDIX B

SUPPLY OF EFFORT IN PARTIAL EQUILIBRIUM

It has been noted in the text that the partial equilibrium effects of income taxation and migration on the supply of effort are equivalent for both sectors in the present context (worker management). Hence the derivation of equations (20) and (21) or (22) and (23) are also very similar. Here we outline the derivation of equations (22) and (23).

1. *Effect of income taxation*

The first-order conditions to the problem given by equations (4) and (5) of the text are:

$$U_1^u - \lambda^u = 0, \tag{B.1}$$

$$U_2^u - \lambda^u(1-t_u)F' = 0, \tag{B.2}$$

and

$$-Z_u + (1-t_u)\left(\frac{F-R}{M}\right) = 0, \tag{B.3}$$

where λ is the Lagrange multiplier. Differentiating equations (B.1)–(B.3) with regard to the tax rate, and dropping the sector subscripts and superscripts, we obtain

$$\begin{bmatrix} U_{11} & U_{12} & -1 \\ U_{21} & \{U_{22}+U_1(1-t)MF''\} & -(1-t)F' \\ -1 & -(1-t)F' & 0 \end{bmatrix} \begin{pmatrix} \dfrac{\partial Z}{\partial t} \\ \dfrac{\partial l}{\partial t} \\ \dfrac{\partial \lambda}{\partial t} \end{pmatrix} = \begin{pmatrix} 0 \\ -U_1F' \\ \left(\dfrac{F-R}{M}\right) \end{pmatrix}. \tag{B.4}$$

From equation (B.4) we have

$$\frac{\partial l}{\partial t} = \frac{1}{D}\left\{U_1F' + \left(\frac{F-R}{M}\right)[U_{11}(1-t)F' - U_{21}]\right\}, \tag{B.5}$$

where

$$D = (-)\left[U_{22} + U_1(1-t)MF'' + \left(\frac{U_2}{U_1}\right)^2 U_{11} - 2\left(\frac{U_2}{U_1}\right)U_{12}\right] > 0$$

is the second-order condition. The first term on the right hand side of (B.5) can be identified as the substitution effect (discouraging the supply of effort, h). The second term, the income effect is, however, ambiguous. Using the first-order conditions in conjunction with equations (A.4) and (A.5) above, equation (B.5) simplifies considerably:

$$\frac{\partial l}{\partial t}=\frac{U_2}{D(1-t)}\left(1-\frac{1}{\sigma_0}\right). \tag{B.6}$$

Clearly,

$$\frac{\partial l}{\partial t}\gtreqless 0 \quad \text{as} \quad \sigma_0 \gtreqless 1. \tag{B.7}$$

Upon further simplification, we obtain

$$\frac{D(1-t)}{U_2}=\frac{1-t}{\sigma_0}\left\{\frac{1+l[\gamma\sigma_0+(\alpha-1)]}{l(1-l)}\right\}$$

or

$$\frac{\partial l}{\partial t}=\frac{l(1-l)(\sigma_0-1)}{(1-t)\{1+l[\gamma\sigma_0+(\alpha-1)]\}}, \tag{B.8}$$

which is stated as equation (22) in the text. Given that $(\alpha-1)<0$, the condition $0<l<1$ suggests that the relationship (B.7) still holds even if $[\gamma\ \sigma_0+(\alpha-1)]$ were to be negative. (Recall that $\alpha\equiv[H\,F'(H)]/(F-R)$, $\gamma\equiv(-)[H\,F''(H)/F'(H)]$.

2. *Effect of migration*

In order to see the effect of migration on the supply of effort by urban workers, we again differentiate the first-order conditions, (B.1) to (B.3), with regard to M. When rearranged, we have

$$\begin{bmatrix} U_{11} & U_{12} & -1 \\ U_{21} & \{U_{22}+(1-t)U_1MF''\} & -(1-t)F' \\ -1 & -(1-t)F' & 0 \end{bmatrix}\begin{pmatrix} \dfrac{\partial Z}{\partial M} \\ \dfrac{\partial l}{\partial M} \\ \dfrac{\partial \lambda}{\partial M} \end{pmatrix}=\begin{pmatrix} 0 \\ U_1(1-t)(1-l)F'' \\ \dfrac{Z}{M}(1-\alpha) \end{pmatrix}. \tag{B.9}$$

Consequently,

$$\frac{\partial l}{\partial M}=\frac{1}{D}\left\{\frac{Z}{M}(1-\alpha)\left[\frac{U_2}{U_1}U_{11}-U_{21}\right]-U_2(1-l)\frac{F''}{F'}\right\}. \tag{B.10}$$

Here again the second term on the right, $-U_2(1-l)F''/F'$, can be identified as the substitution effect caused by a change in M via a change in the return to effort (measured by F'). The first term is the income effect.

As was the case with the effect of taxation, the above term can be simplified considerably using the results stated in Appendix A. First, note

$$\frac{\partial l}{\partial M}=\left(\frac{U_2}{M}\right)\left(\frac{1}{D}\right)(1-\alpha)\left(\frac{\gamma}{1-\alpha}-\frac{1}{\sigma_0}\right). \tag{B.11}$$

But noting that in our model $(1-\alpha)/\gamma$ happens to equal σ, the elasticity of substitution in production, it is evident from equation (B.11) that

$$\frac{\partial l}{\partial M} \gtreqless 0 \quad \text{as} \quad \sigma_0 \gtreqless \sigma. \tag{B.12}$$

Further, an exact quantitative expression may be derived by substituting for D, the second-order condition, in (B.11):

$$\frac{\partial l}{\partial M} = \left(\frac{1}{M}\right)\left\{\frac{l(1-l)[\gamma\sigma_0-(1-\alpha)]}{1+l[\gamma\sigma_0-(1-\alpha)]}\right\}$$

or

$$\frac{\partial l}{\partial M} = \frac{l(1-l)\left(\frac{1}{\sigma}-\frac{1}{\sigma_0}\right)}{M\left[\frac{l}{\sigma}+\frac{1}{\sigma_0}\left(\frac{1}{1-\alpha}-l\right)\right]}. \tag{B.13}$$

Notice that the denominator is positive and, as would be expected, condition (B.12) still holds. This is the desired expression, equation (23) of the text.

APPENDIX C

PROPERTIES OF THE INDIRECT UTILITY FUNCTIONS

Before we undertake the general equilibrium analysis of income taxation, we need to investigate the properties of the indirect utility functions. We are mainly interested in the effects of migration and income taxation on rural and urban utilities.

1. *Income taxation*

Recall that $V^r = U^r(Z^*_r, l^*_r)$, where

$$Z^*_r = \frac{(1-t_r)Q(H^*_r)}{N} \quad \text{and} \quad H^*_r = N(1-l^*_r).$$

We can therefore evaluate

$$V_t^r \equiv \frac{\partial V^r}{\partial t_r} = U_1^r \frac{\partial Z^*_r}{\partial t_r} + U_2^r \frac{\partial l^*_r}{\partial t_r}$$

or, noting that l^*_r also determines Z^*_r, we have

$$V_t^r = (-)U_1^r\left(\frac{Q}{N}\right) + \frac{\partial l^*_r}{\partial t}[U_2^r - (1-t)Q'(H^*_r)U_1^r].$$

However, from the first-order conditions, the terms in square brackets exactly equal zero, and hence

$$V_t^r = (-)U_1^r\left(\frac{Q}{N}\right) < 0. \tag{C.1}$$

Note that the above terms vanish due to a result known as the "envelope theorem" in microeconomic theory.[b] Changes in the income tax rate affect indirect utility

[b] For a discussion of the "envelope theorem," see for example, [21, pp. 327–29].

both directly (by lowering consumption) and indirectly through adjustments in choice of leisure, l^*_r. However, V has already been set at a maximum allowing for adjustments in l^*_r. Hence no further adjustments are possible. It may be further noted that equation (C.1) does not fully capture $\partial Z^*_r/\partial t_r$, as the latter also includes some of the indirect adjustments through l^*_r.

It must now be evident that

$$V_t^u \equiv \frac{\partial V^u}{\partial t_u} = (-)U_1^u(1-l^*_u)Y_u < 0. \tag{C.2}$$

2. *Migration*

Using the above procedures we evaluate

$$V_n^r \equiv \frac{\partial V^r}{\partial N} = U_1^r \frac{\partial Z^*_r}{\partial N} + U_2^r \frac{\partial l^*_r}{\partial N}$$

$$= U_1^r(1-t_r)\left(\frac{H^*Q'-Q}{N^2}\right) + \frac{\partial l^*_r}{\partial N}[U_2^r - (1-t_r)Q'U_1^r]$$

or

$$V_n^r = U_1^r(1-t_r)\left(\frac{H^*Q'-Q}{N^2}\right) < 0 \tag{C.3}$$

as the coefficient of $\partial l^*_r/\partial N$ again vanishes for precisely the reasons given above. It also follows that

$$V_n^u \equiv \frac{\partial V^u}{\partial N} = (1-t_u)\left(\frac{1}{M^2}\right)U_1^u[(F-R)-H^*_uF'] > 0. \tag{C.4}$$

Finally, note that in the alternative model of a profit-maximizing modern sector, although the effects of changes in N are not relevant, the taxation effect is the same as above, qualitatively. [Y_u in equation (C.2) is simply replaced by the wage rate, w].

APPENDIX D

GENERAL EQUILIBRIUM EFFECTS OF AN URBAN INCOME TAX

On the migration pattern, from equation (15), we have

$$\frac{dN}{dt_u} = (-)\frac{V_t^u}{V_n^u - V_n^r} > 0, \tag{D.1}$$

given the preceding discussion on the properties of the indirect utility function.

Urban supply of effort may be written as

$$l^*_u = l_u\{M[N(t_u, t_r)], t_u\}.$$

Hence

$$\frac{dl^*_u}{dt_u} = (-)\frac{\partial l_u}{\partial M}\frac{dN}{dt_u} + \frac{\partial l_u}{\partial t_u}. \tag{D.2}$$

Using equations (B.8) and (B.13), this becomes

$$\frac{dl^*_u}{dt_u}=\frac{(1-\alpha^u)(1-t_u)A^u\Big\{l_u(1-l_u)\Big[\frac{1}{\sigma^u}-\frac{1}{\sigma_0{}^u}\Big]\Big\}+Ml_u(1-l_u)\Big(1-\frac{1}{\sigma_0{}^u}\Big)}{M(1-\alpha^u)(1-t_u)B^u}, \tag{D.3}$$

where we define

$$A^u\equiv(-)\frac{dN}{dt_u}<0$$

and

$$B^u\equiv\Big[\frac{l_u}{\sigma^u}+\frac{1}{\sigma_0{}^u}\Big(\frac{1}{1-\alpha^u}-l_u\Big)\Big]>0.$$

Thus, it is seen that the sign of dl^*_u/dt_u depends on the sign of

$$(1-\alpha^u)(1-t_u)A\Big(\frac{\sigma_0{}^u-\sigma^u}{\sigma^u}\Big)+M(\sigma_0{}^u-1). \tag{D.4}$$

However, for a Cobb-Douglas production function ($\sigma^u=1$), condition (D.4) simplifies into

$$\frac{M(\sigma_0{}^u-1)}{\sigma_0{}^u}\Big[\frac{(-)V_n{}^r}{V_n{}^u-V_n{}^r}\Big] \tag{D.5}$$

which in effect means that the sign of dl^*_u/dt_r is the same as that of

$$(\sigma_0{}^u-1).$$

The peasant's supply-of-effort function reads

$$l^*_r=l_r[N(t_u,t_r),t_r].$$

Hence

$$\frac{dl^*_r}{dt_u}=\frac{\partial l_r}{\partial N}\cdot\frac{dN}{dt_u}$$

or

$$\text{sign }\frac{dl^*_r}{dt_u}=\text{sign }\frac{\partial l_r}{\partial N}$$

using equation (D.1). Further, in view of equation (21) of the text, we have

$$\frac{dl^*_r}{dt_u}\gtreqless 0 \quad\text{as}\quad \sigma_0{}^r\gtreqless\sigma^r. \tag{D.6}$$