THE ONE GAP APPROACH VERSUS THE TWO GAP APPROACH

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THE BASIC LOGIC OF THE "Two Gap Approach" proposed by Chenery and Strout [3] to estimate the potential need for foreign aid is acquiring a concensus of support among economists, but its theoretical implications and the meaning of the practical procedures have not been fully discussed in detail yet. In this paper I would like to develop a "One Gap Approach" and present some comments on the "Two Gap Approach." It will be pointed out that the gap of two gaps is partly attributable to the misspecification of the future growth rate and partly to the different backgrounds, and there is no firm basis to present two point estimates of the two gaps without checking the implicit feasibility. In practical applications, a "One Gap Approach" with various sensitivity analyses can completely replace the "Two Gap Approach" and can deal more clearly with the problem of estimating the necessary self efforts and the necessary foreign needs.

I. ONE GAP APPROACH

We employ the following supply-ceiling type model of a developing economy:[1]

$$Y_{t+1} = Y_t + \frac{1}{k} I_t \tag{1}$$

$$(Y - C)_t = \beta_1 Y_t + \beta_0 \tag{2}$$

$$M_t = \gamma_1 Y_t + \gamma_0 \tag{3}$$

$$E_t = E_0(1 + e)^t \tag{4}$$

$$Y_t = C_t + I_t + E_t - M_t\,. \tag{5}$$

(Y, GNP; C, consumption; I, investment; E, exports; M, imports; t, time).
Then we have the following theorem:

Theorem 1. (i) The future GNP value based on the growth model (1)-(5) can be written as

$$Y_t = \frac{E_0}{k(\lambda - e)}(1 + e)^t + \frac{1}{k\lambda}\left[I_0 - \frac{eE_0}{(\lambda - e)}\right](1 + \lambda)^t - \left(\frac{\beta_0 + \gamma_0}{k\lambda}\right) \tag{6}$$

[1] This is the basic framework of the projection models to estimate the necessary foreign aid adopted by many international organizations and economists (for example, see [3] and [5]). We do not question the basic features of the model, and we will implicitly assume that consistent estimates of the parameters were calculated by an estimation method.

with

$$\lambda \equiv \frac{\beta_1 + \gamma_1}{k} \tag{7}$$

and two initial values of investment and export (I_0 and E_0). (ii) The GNP growth rate ($r_{Y,t} \equiv (Y_{t+1} - Y_t)/Y_t$) is given as a weighted average of two growth rates (λ and e) and will decrease if $\lambda < e(I_0 + E_0)/I_0$, or increase if $\lambda > e(I_0 + E_0)/I_0$. (iii) The import-export gap ($G_{ME,t}$) is given as follows and always equals the investment-saving gap ($G_{IS,t}$)

$$\begin{aligned} G_{ME,t} &\equiv M_t - E_t = E_0\left[\frac{\gamma_1}{k(\lambda - e)} - 1\right](1 + e)^t \\ &+ \frac{\gamma_1}{k\lambda}\left[I_0 - \frac{eE_0}{(\lambda - e)}\right](1 + \lambda)^t + \frac{\beta_1\gamma_0 - \gamma_1\beta_0}{\beta_1 + \gamma_1} \\ &= I_t - [Y - C]_t \equiv G_{IS,t} \,. \end{aligned} \tag{8}$$

(Proof) From (1), (2), (3), and (5):

$$Y_{t+1} - \left(1 + \frac{\beta_1 + \gamma_1}{k}\right)Y_t = \frac{\beta_0 + \gamma_0 - E_0(1 + e)^t}{k}. \tag{9}$$

Here we assume $((\beta_1 + \gamma_1)/k)(\equiv \lambda) \neq e$, then the solution of (9) can be written as

$$Y_t = a_1(1 + e)^t + a_2(1 + \lambda)^t + a_0 \,. \tag{10}$$

Three initial values Y_0, Y_1, and Y_2 are necessary as three coefficients, but these initial values can be replaced by I_0, E_0, and e (assuming $Y_2 > Y_1 > Y_0$, $I_0 > 0$).

$$\begin{pmatrix} a_1 \\ a_2 \\ a_0 \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ (1+e) & (1+\lambda) & 1 \\ (1+e)^2 & (1+\lambda)^2 & 1 \end{bmatrix}^t \begin{pmatrix} Y_0 \\ Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} \dfrac{E_0}{k(\lambda - e)} \\ \dfrac{1}{k\lambda}\left[I_0 - \dfrac{eE_0}{(\lambda - e)}\right] \\ \dfrac{-(\beta_0 - \gamma_0)}{k\lambda} \end{pmatrix}. \tag{11}$$

So we have (6), completing (i). Then we have three cases: Case (1), $\lambda > e(I_0 + E_0)/I_0$. The coefficients of $(1 + e)^t$ and $(1 + \lambda)^t$ in (6) are both positive, and $r_{Y,t}$ will increase to λ in the long run. Case (2), $e(I_0 + E_0)/I_0 > \lambda > e$. The coefficient of $(1 + e)^t$ is positive while that of $(1 + \lambda)^t$ is negative. So $r_{Y,t}$ will decrease until Y_t reaches zero. Case (3), $e > \lambda$. The coefficient of $(1 + e)^t$ is negative while that of $(1 + \lambda)^t$ is positive. The pattern is the same as the case (2), completing (ii). We may call case (1) the case of self-sustained growth because only in this case can economic growth persist infinitely in the future. Part (iii) can be easily verified by the context of the model (1)–(5). (Q.E.D.)

Let us investigate the influence of the parameter changes.[2]

[2] Hereafter we will consider e not as an initial condition but as a parameter because we assume that the export growth rate e ($\equiv E_1/E_0 - 1$) will remain the same in the future.

(i) The influence of a change in the export growth rate (e).

$$\frac{\partial Y_t}{\partial e} = \frac{E_0(1+e)^t}{k(\lambda - e)^2}\left\{1 + \left(\frac{\lambda - e}{1+e}\right)t - \left(\frac{1+\lambda}{1+e}\right)^t\right\}. \tag{12}$$

Then

$$\left.\frac{\partial Y_t}{\partial e}\right|_{t=0} = \left.\frac{\partial Y_t}{\partial e}\right|_{t=1} = 0\,, \quad \left.\frac{\partial Y_t}{\partial e}\right|_{t=2} = -\frac{E_0}{k} < 0\,. \tag{13}$$

$$\lim_{t\to\infty} \frac{\partial Y_t}{\partial e} < 0\,. \tag{14}$$

(ii) The influence of a change in ICOR (k).

$$\frac{\partial Y_t}{\partial k} = \frac{eE_0(1+e)^t}{k^2(\lambda - e)^2} + \frac{(1+\lambda)^2}{k^2}\left\{\frac{t}{(1+\lambda)}\left[\frac{eE_0}{(\lambda - e)} - I_0\right] - \frac{eE_0}{(\lambda - e)^2}\right\}. \tag{15}$$

Then

$$\lim_{t\to\infty} \frac{\partial Y_t}{\partial k} \begin{cases} < 0 & \text{if } \lambda > \left(\dfrac{I_0 + E_0}{I_0}\right)e \\ > 0 & \text{if } \lambda < \left(\dfrac{I_0 + E_0}{I_0}\right)e \end{cases} \tag{16}$$

Especially when $\lambda > e(I_0 + E_0)/I_0$, $\partial Y_t/\partial k$ is negative for all $t \geqq 1$.

(iii) The influence of a change in the marginal propensity to save (β_1). (The influence of the marginal propensity to import [γ_1] is the same).

$$\frac{\partial Y_t}{\partial \beta_1} = \frac{(-)E_0(1+e)^t}{k^2(\lambda - e)^2} + \frac{(1+\lambda)^t}{k^2}\left\{\frac{t}{\lambda(1+\lambda)}\left[I_0 - \frac{eE_0}{(\lambda - e)}\right] + \frac{(2\lambda - e)eE_0}{\lambda^2(\lambda - e)^2} - \frac{eI_0}{\lambda^2}\right\} + \frac{\beta_0 + \gamma_0}{k^2\lambda^2}\,. \tag{17}$$

Then

$$\lim_{t\to\infty} \frac{\partial Y_t}{\partial \beta_1} \begin{cases} > 0 & \text{if } \lambda > \left(\dfrac{I_0 + E_0}{I_0}\right)e \\ < 0 & \text{if } \lambda < \left(\dfrac{I_0 + E_0}{I_0}\right)e \end{cases} . \tag{18}$$

But changes in the parameters are not independent of the initial condition. From (1)-(5) we get

$$k(Y_2 - Y_1) - (\beta_1 + \gamma_1)Y_1 - (\beta_0 - \gamma_0) + E_0(1+e) = 0\,, \tag{19}$$

so when we change the parameters, the variation of the parameter (δe, δk, $\delta\beta_1$, and $\delta\gamma_1$) must satisfy the following relation, treating Y_2, Y_1, and E_0 as constants:

$$(Y_2 - Y_1)\delta k - Y_1(\delta\beta_1 + \delta\gamma_1) + E_0\delta e = 0\,. \tag{20}$$

For example, a change in $e(\delta e)$ must be compensated by a change in k or β_1 (or γ_1). If δe is compensated by δk, the final effect of e compensated by δk is given as

$$\left(\frac{\partial Y}{\partial e}\right)_{\delta k}^{F} \equiv \frac{\partial Y_t}{\partial k}\cdot\frac{\delta k}{\delta e}+\frac{\partial Y_t}{\partial e}\,. \tag{21}$$

When we discuss the effect of a change in the export growth rate, we, must distinguish three definitions: The effect on the current GNP growth rate (which we will call the "impact effect"), the effect on the future average GNP growth rate ($t > 2$) (which we will call the "total effect"), and the effect of the compensated change on the future average GNP growth rate (for $t > 2$) (which we will call the "final effect"). Then we have the following theorem.

Theorem 2. The increase of the export growth rate has (i) a negative impact effect and (ii) a long-run negative total effect, but has (iii) a long-run positive final effect if $\lambda > e(I_0 + E_0)/I_0$.

(Proof) From (9) we have

$$\frac{Y_{t+1}-Y_t}{Y_t}=\frac{\beta_1+\gamma_1}{k}-\frac{\beta_0-\gamma_0}{kY_t}-\frac{E_0(1+e)^t}{Y_t}\,, \tag{22}$$

so $\partial r_{Y,t}/\partial e < 0$, completing (i). Let us define the future average GNP growth rate ($R_{Y,T}$) as

$$R_{Y,t}=\left(\frac{Y_t}{Y_0}\right)^{1/t}-1 \qquad (\text{for } t>2)\,. \tag{23}$$

Then

$$\frac{\partial R_{Y,t}}{\partial e}=\frac{1}{tY_0}\left(\frac{Y_t}{Y_0}\right)^{(1-t)/t}\cdot\frac{\partial Y_t}{\partial e}\,. \tag{24}$$

And from (14) and (24) $\lim_{t\to\infty}\partial R_{Y,t}/\partial e < 0$, completing (ii).

Let us investigate the two cases where δe is compensated by δk and $\delta\beta_1$.

$$\begin{aligned}\left(\frac{\partial Y_t}{\partial e}\right)_{\delta k}^{F} &\equiv \frac{\partial Y_t}{\partial k}\frac{\delta k}{\delta e}+\frac{\partial Y_t}{\partial e}\\ &=\frac{E_0(1+e)^t}{k(\lambda-e)^2}\left\{1+\left(\frac{\lambda-e}{1+\lambda}\right)t-\frac{eE_0}{k(Y_2-Y_1)}\right\}\\ &=\frac{E_0(1+\lambda)^t}{k(\lambda-e)^2}\left\{\frac{t(\lambda-e)^2}{k(Y_2-Y_1)(1-\lambda)}\left[I_0-\frac{eE_0}{(\lambda-e)}\right]\right.\\ &\qquad\left.-\frac{eE_0}{k(Y_2-Y_1)}-1\right\}.\end{aligned} \tag{25}$$

Therefore

$$\lim_{t\to\infty}\left(\frac{\partial Y}{\partial e}\right)_{\delta k}^{F}\begin{cases}>0 & \text{if } \lambda>\left(\dfrac{E_0+I_0}{I_0}\right)e\\ <0 & \text{if } \lambda<\left(\dfrac{E_0+I_0}{I_0}\right)e\end{cases}. \tag{26}$$

$$\left(\frac{\partial Y_t}{\partial e}\right)_{\delta\beta_1}^{F} \equiv \frac{\partial Y_t}{\partial\beta_1}\frac{\delta\beta_1}{\delta e}+\frac{\partial Y_t}{\partial e}$$

$$= \frac{E_0(1+e)^t}{k(\lambda - e)^2}\left\{1 + \left(\frac{\lambda - e}{1+e}\right)t - \frac{E_0}{kY_1}\right\}$$
$$+ \frac{E_0(1+\lambda)^t}{k}\left\{\frac{t}{k\lambda Y_1(1+\lambda)}\left[I_0 - \frac{eE_0}{(\lambda - e)}\right]\right.$$
$$\left. + \frac{(2\lambda - e)eE_0}{k\lambda^2(\lambda - e)^2 Y_1} - \frac{eI_0}{k\lambda^2 Y_1} - \frac{1}{(\lambda - e)^2}\right\} + \frac{(\beta_0 + \gamma_0)E_0}{k^2\lambda^2 Y_1}. \qquad (27)$$

Therefore

$$\lim_{t\to\infty}\left(\frac{\partial Y_t}{\partial e}\right)^F_{\delta\beta_1}\begin{cases} > 0 & \text{if} \quad \lambda > \left(\dfrac{E_0 + I_0}{I_0}\right)e \\ < 0 & \text{if} \quad \lambda < \left(\dfrac{E_0 + I_0}{I_0}\right)e \end{cases}. \qquad (28)$$

It is thus clear that if δe is partly compensated by δk and partly by $\delta\beta_1$ (or $\delta\gamma_1$), the final effect, given by the positive linear combination of (26) and (28), is positive if $\lambda > e(E_0 - I_0)/I_0$, completing (iii). (Q.E.D.)

It is interesting to note that Theorem 2 presents an answer to the paradox of the immiserizing export growth in the framework of the supply-ceiling type model (1)–(5). Some writers, for example Ingram [4], Ball [1], and ECAFE [5, P. 112] pointed out the phenomenon of the immiserizing export growth,[3] the fact that an increase in the export growth rate decreases the GNP growth rate, and cast doubt on the specifications of the model, because the counter-relationship between the two rates of growth is contrary to common understanding. It is now clear that this question refers to the negative impact effect in Theorem 2, but beyond that an increase in the export growth rate also has a negative long-run total effect. On the other hand, an increase in export growth has a positive long-run final effect in the case of self-sustained growth. Thus "immiserizing export growth," or a negative impact effect, does not indicate the misspecification in the model (1)–(5), and an increase in export growth is recommendable in the case of self-sustained growth.

In practical application we can specify various combinations of changing parameters ($e + \delta e$, $k + \delta k$, $\beta_1 + \delta\beta_1$, $\gamma_1 + \delta\gamma_1$), which satisfy the constraint (20) and recalculate the future values of Y_T, $R_{Y,T}$, $G_{ME,T}$, $G_{IS,T}$. We can distinguish two cases: (*a*) simple projection ($\delta e = \delta k = \delta\beta_1 = \delta\gamma_1 = 0$) and (*b*) intentional projection ($|\delta e| + |\delta k| + |\delta\beta_1| + |\delta\gamma_1| > 0$); in the former case, the past structure will persist into the future, and in the latter case, the past structure will be at least partly subject to modification by governmental policies. This procedure of repeating the projections and clarifying the relationship between the sets of parameters and the values of $R_{Y,T}$ and $G_{ME,T}$ may be called the "One Gap Approach," emphasizing the fact that in every projection there exists only one gap.

Here we want to add the following remark. In the framework of the model

3 Of course, the terms of trade effect are eliminated in the context of our model (1)–(5), so the "immiserizing export growth" must be distinguished from Bhagwati's "immiserizing growth" [2].

(1)-(5), two gaps, $G_{ME,t}$ and $G_{IS,t}$, always coincide, i.e., we are treating the two gaps in an ex post facto way. If we collect past data and estimate the parameters by a simultaneous estimation method, we cannot claim that we are estimating the ex-ante parameter or relationship.[4] In the case of intentional projection, we can assume any admissible parameter sets, in the sense that they satisfy the constraint (19), but we cannot claim that these parameters express the ex-ante intentions of micro economic units without further information. We interpret them only as controlled parameters and the intentional projection with these will produce only one ex-post gap, not two.

II. POLITICAL IMPLICATIONS

In this section we will discuss more explicitly the problems of economic policy. We will assume that the two main targets of economic growth are as follows: (*a*) the growth target: to increase the average GNP growth rate ($R_{Y,T}$) in the planning period (T), and (*b*) the balance of payments target: to minimize the balance of payments deficit ($B_T \equiv M_T - E_T$). These two will be specified sometimes as fixed targets and in the other cases as flexible targets.

We assume that the simple projection up to T based on the past structure (e^P, k^P, $\beta_1{}^P$, $\gamma_1{}^P$) produced two values $R^P_{Y,T}$ and $B_T{}^P$, but the government is not satisfied with these and wants to set fixed levels for the two targets: $R^*_{Y,T}$ and B^*_T. Let us express the two targets and the initial condition (19) as follows:

$$R_{Y,T} \equiv \left(\frac{Y_T}{Y_0}\right)^{1/T} - 1 \equiv f(I_0,\ E_0;\ T;\ e,\ k,\ \beta_1,\ \gamma_1) \tag{29}$$

$$B_T \equiv M_T - E_T = I_T - S_T \equiv g(I_0,\ E_0;\ T;\ e,\ k,\ \beta_1,\ \gamma_1) \tag{30}$$

$$h(Y_1,\ Y_2,\ E_0;\ e,\ k,\ \beta_1,\ \gamma_1) = 0\,. \tag{31}$$

Let us specify two (simple and twofold) problems of economic growth:

The Simple-Growth-Problem, which treats $R_{Y,T}$ as a fixed target ($R^*_{Y,T}$) and B_T as a flexible target, seeks the set (e^*, k^*, β^*_1, γ^*_1) such that

$$f(I_0,\ E_0;\ T;\ e^*,\ k^*,\ \beta^*_0,\ \gamma^*_1) = R^*_{Y,T} \tag{32}$$

$$h(Y_1,\ Y_2,\ E_0;\ e^*,\ \beta^*_1,\ \gamma^*_1) = 0 \tag{33}$$

$$e^* > 0,\quad k^* > 0,\quad 1 > \beta^*_1 > 0,\quad 1 > \gamma^*_1 > 0 \qquad (34)^5$$

calculates the necessary variations of the parameters (δ^*_e, δk^*, $\delta\beta^*_1$, $\delta\gamma^*_1$) such that

[4] In some cases we may claim that the ex-post relationship itself indicates directly the ex-ante one, but if we use a simultaneous estimation method we cannot eliminate the influence of the ex-post relations. The adoption of the direct least square cannot be a good replacement. This method may be superior in the theoretical sense that it does not employ information of other ex-post relations, but we cannot avoid the simultaneous equation bias.

[5] Constraint (34) indicates the economic or technical boundary conditions, but they are not the absolute postulates. In some cases, the condition will be less rigid, for example e^* can be negative. And in other cases, the condition will be more rigid, for example $A>e^*$, $B>k^*>C$, $D>\beta^*_1>E$, $F>\gamma^*_1>G$ with certain possible values A, B, C, D, E, F, and G.

$$\delta^*_e \equiv e^* - e^P, \quad \delta k^* \equiv k^* - k^P, \quad \delta\beta^*_1 \equiv \beta^*_1 - \beta_1{}^P, \quad \delta\gamma^*_1 \equiv \gamma^*_1 - \gamma_1{}^P \tag{35}$$

calculates the value of B_T:

$$B^*_T \equiv g(I_0, E_0; T; e^*, k^*, \beta^*_1, \gamma^*_1) . \tag{36}$$

If the difference between $R^*_{Y,T}$ and $R^P_{Y,T}$ is marginal, we have approximately (the suffix P indicates the use of the past structure)

$$R^*_{Y,T} - R^P_{Y,T} = \left(\frac{\partial f}{\partial e}\right)^P \delta e^* + \left(\frac{\partial f}{\partial k}\right)^P \delta k^* + \left(\frac{\partial f}{\partial \beta_1}\right)^P \delta\beta^*_1 + \left(\frac{\partial f}{\partial \gamma_1}\right)^P \delta\gamma^*_1 = \frac{1}{T}(Y_T{}^{1/T-1}\cdot Y_0{}^{-1/T})^P\left[\left(\frac{\partial Y_T}{\partial e}\right)^P \delta e^* + \left(\frac{\partial Y_T}{\partial k}\right)^P \delta k + \left(\frac{\partial Y_T}{\partial \beta_1}\right)^P \delta\beta_1 + \left(\frac{\partial Y_T}{\partial \gamma_1}\right)^P \delta\gamma^*_1\right] \tag{37}$$

$$h(Y_1, Y_2, E_0; \delta e^*, \delta k^*, \delta\beta^*_1, \delta\gamma^*_1) = (Y_2 - Y_1)\delta k^* - Y_1(\delta\beta^*_1 + \delta\gamma^*_1) + E_0\delta e^* = 0 , \tag{38}$$

$$e^P + \delta e^* > 0, \quad k^P + \delta k^* > 0, \quad 1 > \beta_1{}^P + \delta\beta^*_1 > 0, \quad 1 > \gamma_1{}^P + \delta\gamma^*_1 > 0 . \tag{39}$$

Then for any set $(e^*, k^*, \beta^*_1, \gamma^*_1)$ which satisfies (37), (38), and (39) we can calculate

$$B^*_T = (\gamma_1{}^P + \delta\gamma^*_1)Y_0(1 + R^*_{Y,T})^T + \gamma_0 - E_0(1 + e^P + \delta^*_e)^T . \tag{40}$$

We note that the solution of (37), (38), and (39) will be, in general, part of a three-dimensional hyperplane in the four dimensional space of e, k, β_1, and γ_1. Thus the value of the flexible target B^*_T can change within an interval on the real line.

Next let us define the two "Degenerated-Simple-Growth-Problems."

Degenerated-Simple-Growth-Problem (I). The government wants to achieve the fixed target $R^*_{Y,T}$, but with two exogenously determined instruments $\bar{e}^*$ and $\bar{\gamma}^*_1$ (or with fixed $\overline{\delta e}^*$ and $\overline{\delta\gamma}^*_1$).

In this case the original problem will change to seek β^*_1 and k^* such that

$$\left(\frac{\partial Y_T}{\partial k}\right)^P \delta k^* + \left(\frac{\partial Y_T}{\partial \beta_1}\right)^P \delta\beta^*_1 = T(Y_T{}^{1-1/T}\cdot Y_0{}^{1/T})(R^*_{Y,T} - R^P_{Y,T}) - \left(\frac{\partial Y_T}{\partial e}\right)^P \overline{\delta e}^* - \left(\frac{\partial Y_T}{\partial \gamma_1}\right)^P \overline{\delta\gamma}^*_1 \tag{41}$$

$$(Y_2 - Y_1)\delta k^* - Y_1\delta\beta^*_1 = Y_1\overline{\delta\gamma}^*_1 - E_0\overline{\delta e}^* \tag{42}$$

$$k^P + \delta k^* > 0 , \quad 1 > \beta_1{}^P + \delta\beta^*_1 > 0 . \tag{43}$$

In this case the solution space, if one exists, will degenerate to a single point. We can judge the feasibility of this problem by checking whether the solution of (41) and (42) satisfies (43) or not. If the problem is feasible, we can calculate the value of the flexible target as

$$B^*_T = \bar{\gamma}^*_1 Y_0(1 + R^*_{Y,T})^T + \gamma^0 - E_0(1 + \bar{e}^*)^T . \tag{44}$$

It is clear that for a fixed set $(\overline{\delta e^*}, \overline{\delta\gamma^*_1})$ there exists an interval of $R^*_{Y,T}$ to guarantee a feasible solution $(\delta k, \delta\beta_1)$.[6] Thus the possible range of B^*_T constitutes a limited segment outside of which the mechanically calculated value of B^*_T has no meaning.

Degenerated-Simple-Growth-Problem (II). The government wants to achieve the fixed target $R^*_{Y,T}$, but with two exogenously determined instruments $\bar{k}^*$ and $\bar{\beta}^*_1$ (or with fixed $\overline{\delta k^*}$ and $\overline{\delta\beta^*_1}$).

In this case the original problem will change to seek δe^* and $\delta\gamma^*_1$ such that

$$\left(\frac{\partial Y_T}{\partial e}\right)^P \delta e^* + \left(\frac{\partial Y_T}{\partial \gamma_1}\right)^P \delta\gamma^*_1 = T(Y_T^{1-1/T}\cdot Y_0^{1/T})^P(R^*_{Y,T} - R^P_{Y,T}) - \left(\frac{\partial Y_T}{\partial k}\right)^P \overline{\delta k^*} - \left(\frac{\partial Y_T}{\partial \beta_1}\right)^P \overline{\delta\beta^*_1} \qquad (45)$$

$$Y_1\delta\gamma^*_1 - E_0\delta e^* = (Y_2 - Y_1)\overline{\delta k^*} - Y_1\overline{\delta\beta^*_1} \qquad (46)$$

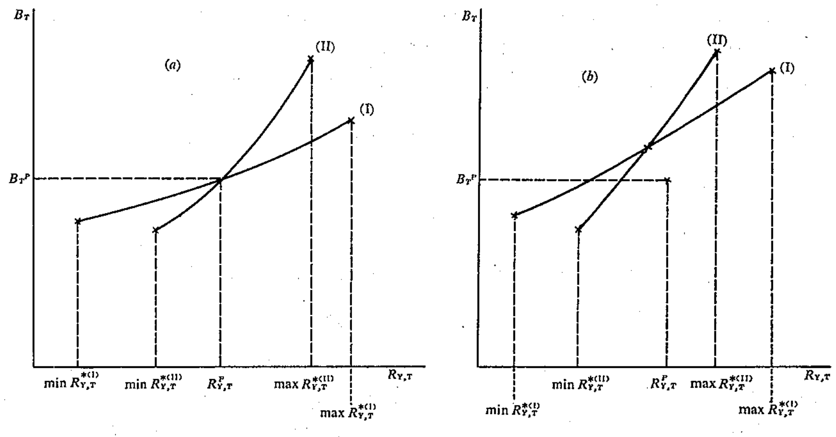
$$e^P + \delta e^* > 0\,, \qquad 1 > \gamma_1{}^P + \delta\gamma^*_1 > 0\,. \qquad (47)$$

The solution, if one exists, is also a single point. And there also exists an interval for $R^*_{Y,T}$ to secure feasibility.

$$B^*_T = (\gamma_1{}^P + \delta\gamma_1)Y_0(1 + R^*_{Y,T})^T + \gamma_0 - E_0(1 + e^P + \delta e^*)^T\,. \qquad (48)$$

B^*_T also constitutes a segment. We remark that in both degenerated problem (I) and (II) two gaps ($G_{ME,T}$ and $G_{IS,T}$) necessarily coincide, so we can interpret B_T as $G_{ME,T}$ or $G_{IS,T}$, though we adopt the form $G_{ME,T}$ for convenience. Figure 1 shows the possible intervals of $R_{Y,T}$, $(\min R^{*(I)}_{Y,T}, \max R^{*(I)}_{Y,T})$ in the degenerated problem (I) and $(\min R^{*(II)}_{Y,T}, \max R^{*(II)}_{Y,T})$ in (II) as well as the corresponding ranges of B_T. Figure 1 (*a*) shows the case in which $e^* = e^P$, $\gamma^*_1 = \gamma_1{}^P$ or $k^* = k^P$, $\beta^*_1 = \beta_1{}^P$, and the two solution curves will pass the point

Fig. 1. The Long-run Relation $(\lambda > e(I_0 + E_0)/I_0)$



6 If $(R^*_{Y,T} - R^P_{Y,T})$ cannot be interpreted as a marginal change, we always can refer to the original formula (6) and recalculate the average slope instead of a partical derivative such as $(\partial f/\partial k)^P$.

$(R^P_{Y,T},\ B_T{}^P)$ when $\delta k^* = 0$, $\delta\beta^*_1 = 0$, or $\delta e^* = 0$, $\delta\gamma^*_1 = 0$. Figure 1 (*b*) shows the case in which $e^* \neq e^P$, $\gamma^*_1 \neq \gamma_1{}^P$ or $k^* \neq k^P$, $\beta^*_1 \neq \beta_1{}^P$. We can interpret the existence of the feasible ranges of $R^*_{Y,T}$ in the following way. Reinterpreting the boundary conditions more realistically (see footnote 5), the possible ranges of $R^*_{Y,T}$ will indicate the range of the attainable growth rate by maximum self-effort. But when a certain level of $R^*_{Y,T}$ is achieved by a certain amount of self-help, there still remains a certain balance of payments deficit. Thus the point $(R_{Y,T},\ B_T)$ in Figure 1 (*a*) and Figure 1 (*b*) with the implicit set of parameters $(e^*,\ k^*,\ \beta^*_1,\ \gamma^*_1)$ indicates (i) the attainable $R^*_{Y,T}$ and the corresponding self-effort $(e^* - e^p,\ k^* - k^p,\ \beta^*_1 - \beta_1{}^p,\ \gamma^*_1 - \gamma_1{}^p)$ and the corresponding need for foreign aid (B^T), and also (ii) the limited range of economic growth by maximum self-effort.

Now we investigate the long-run characteristics of the curves (I) and (II). First, in the degenerated problem (I),

$$\lim_{t\to\infty}\left(\frac{\partial Y_t}{\partial\beta_1}\right)^F_{\delta k} = \lim_{t\to\infty}\left\{\left(\frac{Y_1}{Y_2 - Y_1}\right)\left(\frac{\partial Y_t}{\partial k}\right) + \left(\frac{\partial Y_t}{\partial\beta_1}\right)^P\right\}$$
$$\doteqdot \frac{(1+\lambda)^{t-1}t}{k^2}\left[I_0 - \frac{eE_0}{(\lambda - e)}\right]\left[\frac{Y_2 - (1+\lambda)Y_1}{\lambda(Y_2 - Y_1)}\right]. \tag{49}$$

When $\lambda > e(I_0 + E_0)/I_0$, the GNP growth rate will increase to λ; therefore $1 < Y_2/Y_1 < 1 + \lambda$ and

$$\lim_{t\to\infty}\left(\frac{\partial Y_T}{\partial\beta_1}\right)^F_{\delta k} < 0\,. \tag{50}$$

Thus the necessary parameter variations to increase the long-run growth rate are negative ($\delta k < 0$ and $\delta\beta_1 < 0$), and B_T will increase by

$$\left(\frac{\partial B_T}{\partial\beta_1}\right)^F_{\delta k} = \gamma_1{}^P\left(\frac{\partial Y_t}{\partial\beta_1}\right)^F_{\delta k}. \tag{51}$$

Secondly in the degenerated problem (II),

$$\lim_{t\to\infty}\left(\frac{\partial Y_t}{\partial e}\right)^F_{\delta\gamma_1} = \lim_{t\to\infty}\left\{\left(\frac{E_0}{Y_1}\right)\left(\frac{\partial Y_t}{\partial\gamma_1}\right)^P + \left(\frac{\partial Y_t}{\partial e}\right)^P\right\}$$
$$\doteqdot \frac{E_0(1+\lambda)^{t-1}\cdot t}{k^2\lambda Y_1}\left[I_0 - \frac{eE_0}{(\lambda - e)}\right]. \tag{52}$$

When $\lambda > e(I_0+E_0)/I_0$,

$$\lim_{t\to\infty}\left(\frac{\partial Y_t}{\partial e}\right)^F_{\delta\gamma_1} > 0\,. \tag{53}$$

So the necessary parameter variations are positive ($\delta e > 0$ and $\delta\gamma_1 > 0$). In this case

$$\lim_{t\to\infty}\left(\frac{\partial B_T}{\partial e}\right)^F_{\delta\gamma_1} = \lim_{t\to\infty}\left\{\gamma_1\left(\frac{\partial Y_T}{\partial e}\right)^F_{\delta\gamma_1} + \left(\frac{\delta\gamma}{\delta e}\right)Y_T{}^P - E_0(1+e)^{t-1}\cdot t\right\}. \tag{54}$$

But if $\lambda > e(I_0 + E_0)/I_0$,

$$\lim_{t\to\infty}\left\{\left(\frac{\delta\gamma}{\delta e}\right)Y_T{}^P - E_0(1+e)^{t-1}t\right\} = \lim_{t\to\infty}\left\{\left(\frac{E_0}{Y_1}\right)Y_T{}^P - E_0(1+e)^{t-1}t\right\} > 0\,. \quad (55)$$

So if we assume $(\partial Y_T/\partial\beta_1)^F_{\delta k} = (\partial Y_T/\partial e)^F_{\delta\gamma}$, e.g., if the average GNP growth rate is increased to the same extent by the variations $(\delta k,\ \delta\beta_1)$ and $(\delta e,\ \delta\gamma_1)$, then B_T will increase more rapidly when $(\delta e,\ \delta\gamma_1)$. This qualitative comparison will be maintained if we take a sufficiently long period. Figures 1 (*a*) and (*b*) show the comparison in this self-sustained case, $\lambda > e(I_0 + E_0)/I_0$.

Now we turn to the next problem:

The Twofold-Growth Problem, which treats $R_{Y,T}$ and B_T as two fixed targets $(R^*_{Y,T}$ and $B^*_T)$, seeks the set $(e^*,\ k^*,\ \beta^*_1,\ \gamma^*_1)$ such that

$$f(I_0,\ E_0;\ T;\ e^*,\ k^*,\ \beta^*_1,\ \gamma^*_1) = R^*_{Y,T} \quad (56)$$

$$g(I_0,\ E_0;\ T;\ e^*,\ k^*,\ \beta^*_1,\ \gamma^*_1) = B^*_T \quad (57)$$

$$h(Y_1,\ Y_2,\ E_0;\ e^*,\ k^*,\ \beta^*_1,\ \gamma^*_1) = 0 \quad (58)$$

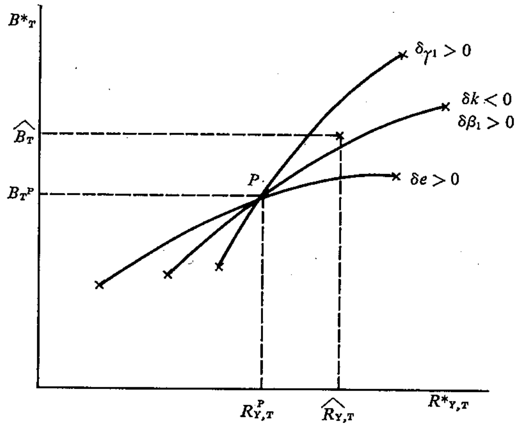
$$e^* > 0\,,\quad k^* > 0,\quad 1 > \beta^*_1 > 0\,,\quad 1 > \gamma^*_1 > 0 \quad (59)$$

and calculates the necessary parameter variations $(\delta e^*,\ \delta k^*,\ \delta\beta^*_1,\ \delta\gamma^*_1)$ such that

$$\delta e^* \equiv e^* - e^P\,,\quad \delta k^* \equiv k^* - k^P\,,\quad \delta\beta^*_1 \equiv \beta^*_1 - \beta_1{}^P\,,\quad \delta\gamma^*_1 \equiv \gamma^*_1 - \gamma_1{}^P\,. \quad (60)$$

If we treat the problem in the form of an approximate linear expansion, the solution space will be, in general, a linear segment in the four dimensional space of e, k, β_1, and γ_1. Now from (14), (16), and (18) an increase in the average rate of growth in the long-run necessitates $\delta e > 0$ or $\delta k < 0$ or $\delta\beta_1 > 0$ or $\delta\gamma_1 > 0$ if we treat them independently and neglect the initial condition (20). So the independent parameter change will result in four curves as for example those in Figure 2, considering the adequate boundary conditions. (Therefore each curve is cut from above and from below by these restraints). This means that there exists an infinite set of parameters to achieve the fixed

Fig. 2.



target; for example, to achieve $(\hat{R}_{Y,T}, \hat{B}_T)$ we can select a combination of $\delta\gamma_1$ (>0) and $\delta e(>0)$ or $\delta\gamma_1(>0)$ and $\delta k(<0)$ or $\delta\gamma_1(>0)$ and $\delta\beta_1(>0)$, or $\delta\gamma_1(>0)$, $\delta\beta_1(>0)$ and $\delta e(>0)$, etc.

We state the following theorem for the convenience of the following discussion.

Theorem 3. We specify the two Degenerated-Simple-Growth-Problem with the specified instruments $(e^p, \gamma_1{}^p, k^p, \beta_1{}^p)$: Problem (I) to seek δk^* and $\delta\beta^*{}_1$ with predetermined e^* and $\gamma^*{}_1$, and Problem (II) to seek δe^* and $\delta\gamma^*{}_1$ with predetermined k^* and $\beta^*{}_1$. Then in the common region of the feasible intervals of $R_{Y,T}$, the value of B_T is larger (smaller) in problem (II) than in problem (I) if $R^*{}_{Y,T} > R^p_{Y,T}(R^*{}_{Y,T} < R^p_{Y,T})$ and if $\lambda \gtreqless e(I_0+E_0)/I_0$.

This theorem states that in the interesting case of self-sustained growth the estimated balance of payments deficit will be larger in problem (II) if the government sets the relatively ambitious target $R^*{}_{Y,T} > R^p_{Y,T}$. The point is that the implicit machanism between targets and instruments which was stated in the Twofold-Growth-Problem in a general way must be clearly understood, and the two estimates of $B^*{}_T$ for the same $R^*{}_{Y,T}$ can be quite different in two different specifications of the problem, (I) and (II), while there always exists only one gap instead of two, and the difference between these two estimates is a completely independent matter from the gap between ex-ante and ex-post gaps.

III. TWO GAP APPROACH

Let us define the Two Gap Approach as follows:

Procedure of the Two Gap Approach: (i) On the basis of the supply-ceiling type model (1)–(5), define the import-export gap (G_1) and the investment-saving gap (G_2)[7] as follows:

$$G_{1,T} \equiv \gamma_1 Y_0(1+R_{Y,T})^T + \gamma_0 - E_0(1+e)^T \tag{61}$$

$$G_{2,T} \equiv kY_0(1+R_{Y,T})^T R_{Y,T} - \beta_1 Y_0(1+R_{Y,T})^T \beta_0\,. \tag{62}$$

(ii) Define the dominant gap (G_D) as

$$G_{D,T} \equiv \max(G_{1,T},\ G_{2,T}) \tag{63}$$

and interpret this $G_{D,T}$ as the necessary amount of foreign aid to achieve $R_{Y,T}$.

First we note the special implicit assumptions about the pattern of the rate of growth of (61) and (62). (62) assumes that $r_{Y,T}(\equiv (Y_{T+1}-Y_T)/Y_T)$ equals $R_{Y,T}$, i.e., a constant rate of growth $(R_{Y,T})$ prevails in every period $t(2 \leq T)$. From an understanding of the future dynamic path of the model (6), this constant-rate-of-growth assumption requires very complicated successive parameter variations. Originally, the purpose of the gap estimate is to estimate the gap to achieve a certain average rate of growth, which is not necessarily constant over time. There will be a great difference in the governmental burden in the two policies: (1) to achieve an average rate of growth and (ii) to maintain

[7] We adopt the special symbols G_1 and G_2 to distinguish them from G_{ME} and G_{SI}; the latter two coincide by definition, but the former two do not necessarily coincide.

a constant rate of growth. In (1) the government has only one target of growth, while in (2) it is faced with the problem of stability as well as growth. So we must say that (61) and (62) are overspecifications of the original problem.[8] This overspecification causes a practical difficulty. If we define

$$G_{1,T} \equiv \gamma_1 Y_0(1+R_{Y,T})^T + \gamma_0 - E_0(1+e)^T \tag{64}$$

$$G_{2,T} \equiv kY_0(1+R_{Y,T})^T \cdot \left(\frac{Y_{T+1}-Y_T}{Y_T}\right) - \beta_1 Y_0(1+R_{Y,T})^T - \beta_0 \tag{65}$$

where Y_T is defined as (6) and $R_{Y,T}$ as (23), it always holds that

$$G_{1,T} = G_{2,T}\,, \tag{66}$$

but if we define $r_{Y,T} = R_{Y,T}$, then it always holds that

$$G_{1,T} \neq G_{2,T}\,. \tag{67}$$

Thus if the purpose is to assess the aid needed to achieve an average rate of growth, $r_{Y,T} = R_{Y,T}$ is a technical misspecification which produces a misspecification error. So let us interpret pro tempore (61) and (62) as (64) and (65) and proceed to the next step.

The second comment concerns the lack of an explicit check of the feasibility of the policy problem. In the two gap approach the government adopts the past or modified parameter values (γ_1, e, k, β_1) to calculate (64) and (65). So in our terminology the two gap approach is dealing with the two Degenerated-Simple-Growth-problems. We must therefore recognize the limited feasibility of the problem (see Figure 1 for example) and the mechanical repetition of the calculations assuming any average rate of growth is a risky procedure, neglecting the limits of self-effort. The assumed average rate of growth must be less than $\max R^{*(\mathrm{I})}_{Y,T}$ or $\max R^{*(\mathrm{II})}_{Y,T}$, otherwise the economic effect of the foreign aid (B_T) will be unreasonably highly evaluated to produce an erroneous impression.[9] In this sense, the sensitivity analysis (estimates of $\partial Y_T/\partial e$, $\partial Y_T/\partial k$, $\partial Y_T/\partial \beta_1$, $\partial Y_T/\partial \gamma_1$) and the solution of the two degenerated problems are more meaningful even if they require far more effort and calculations.

Thirdly, we must be aware of the existence of the divergence between two specifications of the degenerated problems (see Theorem 3 and Figure 1) and the fact that this divergence (or "the gap in the two gaps") is consistent with the correct specification of the problem ((6), (23), and the two degenerated problems). From this, we can say that a "gap between the two gaps" clearly exists in the context of our model (1)–(5). But the point is that "the gap between the two

8 (i) The assumption of constancy cannot be held because, practically, $Y_2/Y_1 \neq Y_1/Y_0 \neq R_{Y,T}$.
(ii) In practical applications, when $R^{\mathrm{I}}_{Y,T} \neq R^{*}_{Y,T}$, the projected rate of growth is often set in an increasing trend from $R_{Y,T}$ to $R^{*}_{Y,T}$ in the beginning of the projection period. This fact or convention means that the assumption of constancy is not of political importance, but is made only for convenience.

9 In the extreme case there exists the case of empty feasible solutions if the government adopts a special combination of the pre-specified parameter values and the average rate of growth. Then all the results of the estimates are meaningless.

gaps" is due to the different self-efforts and is not related to the difference between the ex-ante and ex-post divergence, so that there exists only one gap in each intentional projection.

IV. SUMMARY AND CONCLUSIONS

In this paper we used a model (1)–(5) and developed a one-gap-approach which we critically compared to the two-gap approach. In the writer's judgment, the latter has the relative advantage of simplicity and convenience in specification and estimation, but the loss due to overspecification of the problem and the neglect of feasibility is greater than these merits.

Of course, the simple scheme of our model (1)–(5) contains another misspecification and lack,[10] and is naturally subject to further examination and analysis.

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[10] The writer wants to point out the misspecification of the production function in the model (1)–(5) and the explicit treatment of the terms of trade effect in the other notes.