

OPTIMAL STRATEGIES FOR DEVELOPMENT: THE CASE OF INDIA

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I. INTRODUCTION

ONE OF THE MAIN features of economic growth in developing countries is to finance a higher level of investment by using external resources. However, in many developing countries, to expect more external resources is very difficult. The causes of difficulties depend on either economic or political situations and there are some indications for improvement although not decisive. However, to rapidly generate a self-sustained growth situation in developing countries is earnestly desired by both the developing countries and the developed countries. Now, our objective in the present study is to find a possibility for a more effective strategy of development with the less requirement for external resources.

There has been relatively little theoretical study to explore the same objective that we are attempting in the present study. What we are going to do is to investigate some characteristics of optimal growth paths over a period of time. Some main points in the existing optimal growth theory are that they (1) assume a closed economy, (2) focus mainly on the allocation of resources between investment goods and consumption goods production, and (3) study long-run equilibrium conditions of the optimal growth path.¹ However, in the present study, we have (1) to assume an open economy, (2) to focus mainly on the role of external resources in take-off process, and (3) to study optimum take-off conditions for generating a self-sustained growth situation within a finite time horizon.² For the purpose, this study explores the properties of optimal strategy for take-off process: the optimal strategy is determined under conditions to minimize a required time for generating self-sustained and balanced growth situation.

While the formulation of the problem is designed to bring out its general features, the resulting programming model is applied to the Indian planning situation described in the Long Term Perspective for 1967-81.³ India was chosen for the present study because it receives substantial external resources and also it has an explicitly formulated long-term perspective for future development. And

¹ We refer these properties to [2]. And, despite these properties, we have gotten considerable insight into the present problem from [11], [4], [1], and [12].

² [2] gives us a lot of suggestions for the present study.

³ See [7].

the Indian economy represents a typical situation from which many developing countries are attempting to start a process of accelerated growth. The application of the numerical data in Indian long-term perspective to our programming model generates an optimal take-off process which we refer to as "the basic solution." At the same time, we considered some alternative strategies by which some possibilities to develop more effective strategies are examined. In respect to the alternative strategies, we used some plan documents and other analysis of the Indian economy.⁴

II. THEORETICAL FRAMEWORK

1. *The Model of Growth*

The problem of determining an optimal time pattern of net capital inflow for taking-off into self-sustained and balanced growth situation will now be stated according to the Pontryagin's Maximum Principle.⁵ The objective is to minimize the time required for the take-off process. The constraints are the self-sustained and balanced growth situation to be attained at the end of the take-off process and the definitional, structural and behavioral relationships on the take-off process. Variables and parameters are defined as follows.

Variables:

V = net domestic product (NDP),

I = net investment,

S = domestic savings,

F = net capital inflow from abroad,

M = demand for traditional imports,

M^* = demand for imports adjusted on take-off process,

E = exports,

C = consumption.

Parameters:

λ = rate of growth of investment,

$\bar{\lambda}$ = upper limit of rate of growth of investment,

β = investment productivity,

s = marginal rate of savings,

α = average rate of savings,

ϵ = rate of growth of exports,

m = imports ratio,

n = rate of growth of population,

T = terminal year of take-off process.

⁴ For the purpose, we have used the data given in [5, 6, 7], [10], and [8].

⁵ See [9].

Let us consider the economic structure of developing country in the following two stages: (a) in take-off process and (b) in post take-off process that is in a self-sustained and balanced growth situation.

(a) *Take-off Process*:⁶ The economic structure in take-off process is stated as follows. Investment is financed by domestic savings and net capital inflow from abroad.

$$I = S + F. \quad (1)$$

This investment can grow at λ ,

$$\dot{I} = \lambda I, \quad (2)$$

where the rate of growth of investment can take the value within the following region.

$$0 \leq \lambda \leq \bar{\lambda}. \quad (3)$$

The upper limit of this rate of growth of investment depends mainly on the limit of the ability to invest, which reflects the concept that absorptive capacity for additional investment at any point of time is limited by the supply of complementary inputs. For the level of investment as given above, the level of net domestic product can increase in the following way,

$$\dot{V} = \beta I, \quad (4)$$

where β stands for incremental output-capital ratio. We refer to this parameter β as "investment productivity," and assume the constant values through our arguments in the present study. Increase in domestic savings is a certain fraction of increase in net domestic product.

$$\dot{S} = s\dot{V}. \quad (5)$$

This sort of saving function is assuming constant marginal rate of savings. Minimum imports is required to sustain a given level of NDP. In this respect, the part of imports is a certain fraction of NDP.

$$M = m_0 V, \quad (6)$$

where m_0 stands for the value of import ratio at the base year. And this sort of imports results from the traditional import structure of the economy concerned, by which we mean the import structure to remain unchanged from the base year. So we refer to this import requirement as "traditional imports." However, as will be stated later, actual import requirement on the take-off process may exceed this minimum level. Exports grow at ϵ .

$$\dot{E} = \epsilon E. \quad (7)$$

This rate of growth of exports is largely limited by demand conditions in the world market concerned. A rapid increase in exports requires the development of new export products, which is limited by productive capacity as well as organizational and institutional factors in the developing countries. We assume that the effects of government policies to increase exports are summarized by the rate of

⁶ As to the concept of "take-off process," we have received a lot of suggestions from [3]. But, to close the gap between the saving-gap and the trade-gap, we depend on [2].

growth of exports.

As stated above, the net capital inflow from abroad is required to finance a shortage of investment over domestic savings,

$$F = I - S > 0.$$

And, when the required amount of the net capital inflow exceeds the trade gap at the same moment of time,

$$B = F - (M - E) > 0, \quad (8)$$

actual import requirement at the moment should be

$$M^* = M + B. \quad (9)$$

Otherwise, we cannot expect the scheduled investment growth at that growth rate λ to be maintained. Thus we have the balance equation on the national account,

$$V = C + I + E - M^*. \quad (10)$$

We treat consumption to be the residual in the equation (10). However, when the growth of consumption conflicts with the constraint

$$\hat{C} \geq nC, \quad (11)$$

this constraint becomes effective in its equality. This means that the per-capita consumption should, at least, not be declined.

(b) *The Self-sustained and Balanced Growth Process:* Now let us consider the economic structure at the stage of post take-off process after completing the take-off into self-sustained and balanced growth situation. In respect of the self-sustained growth, the net capital inflow from abroad naturally becomes zero,

$$F = 0. \quad (12)$$

And also, in respect of the balanced growth, rates of growth of NDP, investment, domestic savings, consumption, imports and exports become equal to the balanced rate of growth

$$\hat{V} = \hat{C} = \hat{S} = \hat{I} = \hat{M} = \hat{E} = g^*, \quad (13)$$

where g^* stands for the balanced rate of growth. In these respects, the economic structure on the self-sustained and balanced growth situation is stated, together with relations (12) and (13), by the following relations,

$$I = S, \quad (1^*)$$

$$\hat{V} = \beta I, \quad (4)$$

$$S = \alpha^* V, \quad (5^*)$$

$$M = m^* V, \quad (6^*)$$

$$B = 0, \quad (8^*)$$

$$V = C + I + E - M. \quad (10^*)$$

α^* is an average rate of savings required for maintaining the balanced rate of growth under the given level of investment productivity. And m^* is an import ratio standing for a desired degree of the economy to depend on imports in the self-sustained and balanced growth situation.

We suppose that the take-off process is terminated after some time T . This can be stated

$$\begin{aligned}\dot{I} &= aI - \dot{E}, \\ \dot{E} &= \varepsilon E, \\ \dot{V} &= \beta I,\end{aligned}\tag{14}$$

for $t \geq T$, where

$$a = (\alpha^* + m^*)\beta.\tag{15}$$

This system has the solution

$$I(t) = [I_T - AE_T]e^{a(t-T)} + AE_Te^{\varepsilon(t-T)},\tag{16}$$

$$V(t) = [V_T - BE_T]e^{a(t-T)} + BE_Te^{\varepsilon(t-T)},\tag{17}$$

$$E(t) = E_Te^{\varepsilon(t-T)},\tag{18}$$

where

$$E_T = E_0e^{\varepsilon T},\tag{19}$$

$$A = \frac{\varepsilon}{a - \varepsilon},\tag{20}$$

$$B = \frac{\beta}{a - \varepsilon}.\tag{21}$$

These relations verify that the balanced growth is not possible for arbitrary values of I_T and V_T . The balanced growth process is realized only when

$$I^*_T = AE_T,\tag{22}$$

$$V^*_T = BE_T.\tag{23}$$

Then the economy is expanding at the rate of growth of exports. Thus the balanced rate of growth is equal to the rate of growth of exports,

$$g^* = \varepsilon.\tag{24}$$

On the balanced growth process, the average rate of savings is maintained at certain level, $\alpha^* = I_T/V_T$. Then we have

$$\varepsilon = \alpha^*\beta.\tag{25}$$

2. Termination of the Take-off Process in Minimum Time

As stated in the preceding section, the take-off process means the growth process of the economy to start from the given initial situation $[V_0, I_0]$ shifts to the specified terminal situation $[V^*_T, I^*_T]$. We consider an optimization of this take-off process to terminate it in minimum time horizon.

This optimization problem is mathematically formulated as follows:

To find the time pattern

$$\lambda(t), \quad 0 \leq t \leq T\tag{26}$$

to minimize the functional

$$\int_0^T dt\tag{27}$$

subject to

$$\begin{aligned}
 \dot{V} &= \beta I, \\
 \dot{I} &= \lambda I, \\
 0 &\leq \lambda \leq \bar{\lambda}, \\
 V_0 \text{ and } I_0 &= \text{given}, \\
 V_T &= \frac{\beta E_0 e^{\epsilon T}}{a - \epsilon}, \\
 I_T &= \frac{\epsilon E_0 e^{\epsilon T}}{a - \epsilon}.
 \end{aligned} \tag{28}$$

As the economic meaning, this problem is to find the time pattern of the rate of growth of investment in order to terminate the take-off process in minimum time.

The immediate solution of the problem mentioned above gives (i) the time pattern of growth rate of investment for $0 \leq t \leq T$ [$\lambda(t)$], (ii) the terminal levels of net domestic product and investment [V^*_T, I^*_T], (iii) the time patterns of net domestic product and investment for $0 \leq t \leq T$ [$V(t), I(t)$], and (iv) the terminal year of the take-off process [T]. However, using these results, we have (v) the required marginal rate of savings [s], (vi) the time patterns of domestic savings and consumption [$S(t), C(t)$], (vii) the time pattern of traditional imports [$M(t)$], and (viii) the time pattern of net capital inflow from abroad [$F(t)$]. Apart from these items, (ix) the time pattern of exports [$E(t)$] is given exogenously. Thus we have (x) the time pattern of actually required imports on take-off process [$M^*(t)$], which we refer to as "adjusted imports."

To avoid distinguishing many cases in the mathematical solution, we will use, whenever we need it, the numerical values of variables and parameters described in the Indian plan documents and other analysis of Indian economy. These values are shown in Tables I and II.

3. Optimal Time Pattern of the Rate of Growth of Investment

To determine an optimal solution for the problem stated above which is to terminate the take-off process in the minimum time horizon, we use the Pontryagin's Maximum Principle.

Introducing two auxiliary variables $p_1(t)$ and $p_2(t)$ which are corresponding to net domestic product $V(t)$ and investment $I(t)$ respectively, we have the following Hamiltonian expression:

$$\begin{aligned}
 H &= -1 + p_1 \dot{V} + p_2 \dot{I}, \\
 &= -1 + (p_1 \beta + p_2 \lambda) I.
 \end{aligned} \tag{29}$$

The necessary conditions for a time pattern of the rate of growth of investment $\lambda^*(t)$ to be optimal are:

- (a) The control variable λ is chosen at value λ^* so as to maximize H subject to conditions (28).
- (b) There exists continuous functions $p_1(t)$ and $p_2(t)$ which, together with the

continuous functions $V(t)$ and $I(t)$, satisfy the following canonical differential equations

$$\begin{aligned}\dot{V} &= \beta I, \\ \dot{I} &= \lambda I, \\ \dot{p}_1 &= 0, \end{aligned} \tag{30}$$

$$\dot{p}_2 = -p_1\beta - p_2\lambda. \tag{31}$$

(c) If $\lambda^*(t)$ is optimal, maximum value of H is zero through $0 \leq t \leq T$,

$$\max. H = 0. \tag{32}$$

In the Hamiltonian H (29), the control variable is λ . Then, in order to maximize H , λ has to take the value as large as possible for $p_2(t) > 0$, and to take it as small as possible for $p_2(t) < 0$.

Equation (30) has the solution

$$p_1(t) = p^*_1 \quad \text{for} \quad 0 \leq t \leq T. \tag{33}$$

Thus we treat $p_1(t)$ as constant through $0 \leq t \leq T$.

[Case I]

Let us consider the case that $p_2(0) > 0$. Then, $\lambda(0) = \lambda^* > 0$ and, as long as $p_2(t) > 0$, equation (31) generates the solution

$$p_2(t) = \left[p_2(0) + \frac{p^*_1\beta}{\lambda^*} \right] e^{-\lambda^*t} - \frac{p^*_1\beta}{\lambda^*}. \tag{34}$$

If $p^*_1 \leq 0$, $p_2(t)$ remains positive and $\lambda(t) = \lambda^*$. However, if $p^*_1 > 0$, there exists some time $\tau > 0$ such that

$$p_2(\tau) = 0,$$

$$\dot{p}_2(\tau) < 0,$$

so that, for η sufficiently small,

$$\lambda(\tau + \eta) = 0.$$

Then, equation (31) has the solution, for $\tau < t$

$$p_2(t) = -p^*_1\beta(t - \tau), \tag{35}$$

so that $p_2(t)$ remains negative for $\tau < t$.

Thus, for $p^*_1 > 0$,

$$\lambda(t) = \lambda^* \quad \text{for} \quad 0 \leq t \leq \tau, \tag{36}$$

$$\lambda(t) = 0 \quad \text{for} \quad \tau \leq t \leq T.$$

And these do not contradict to the other optimality condition (32).

[Case II]

Next, let us consider the case such that $p_2(0) < 0$. Then, $\lambda(0) = 0$ and, as long as $p_2(t) < 0$, equation (31) generates the solution

$$p_2(t) = -p^*_1\beta t + p_2(0). \tag{37}$$

If $p^*_1 \geq 0$, $p_2(t)$ remains negative and $\lambda(t) = 0$ through $0 \leq t \leq T$. This is not actually feasible, because this means zero rate of growth of investment over take-off period. So we neglect this case in the following arguments.

If $p^*_1 < 0$, there exists some time $\tau > 0$ such that

$$\begin{aligned} p_2(\tau) &= 0, \\ \dot{p}_2(\tau) &> 0, \end{aligned}$$

so that, for η sufficiently small,

$$\lambda(\tau + \eta) = \lambda^*.$$

Then equation (31) has the solution, for $\tau < t$,

$$p_2(t) = \frac{p^*_1 \beta}{\lambda^*} [1 - e^{-\lambda^*(t-\tau)}], \quad (38)$$

so that $p_2(t)$ remains positive for $\tau < t$.

Thus, for $p^*_1 < 0$,

$$\begin{aligned} \lambda(t) &= 0 & \text{for } 0 \leq t \leq \tau, \\ \lambda(t) &= \lambda^* & \text{for } \tau \leq t \leq T. \end{aligned} \quad (39)$$

However, this case [$p_2(0) < 0$, $p^*_1 < 0$] contradicts with the other optimality condition (32). There does not exist the optimal solution in this case.

[Case III]

In the case such that $p_2(0) = 0$,

$$\dot{p}_2(0) = -p^*_1 \beta.$$

Since p^*_1 cannot be equal to zero in this case (otherwise the optimality condition (32) is not satisfied), $\dot{p}_2(0)$ is different from zero and $p_2(\eta)$, for sufficiently small η , is either positive or negative. Then one of the previous cases becomes true.

Thus, we will consider only Case I in the following arguments. Now let us examine the optimality of this case in respect to the terminal conditions. For the purpose, we compute the corresponding values of V_τ and I_τ . In the first phase, for $0 \leq t \leq \tau$, $\lambda(t) = \lambda^*$, so that

$$\begin{aligned} I(t) &= I_0 e^{\lambda^* t}, \\ V(t) &= V_0 + \frac{\beta}{\lambda^*} I_0 [e^{\lambda^* t} - 1]. \end{aligned}$$

In the second phase, for $\tau \leq t \leq T$, $\lambda(t) = 0$, so that

$$\begin{aligned} I(t) &= I_0 e^{\lambda^* \tau}, \\ V(t) &= \left[V_0 - \frac{\beta}{\lambda^*} I_0 \right] + \beta I_0 e^{\lambda^* \tau} \left[\frac{1}{\lambda^*} + T - \tau \right]. \end{aligned}$$

Therefore, the terminal conditions are

$$\begin{aligned} I_0 e^{\lambda^* \tau} &= \frac{\varepsilon}{a - \varepsilon} E_0 e^{\varepsilon T}, \\ \left[V_0 - \frac{\beta}{\lambda^*} I_0 \right] + \beta I_0 e^{\lambda^* \tau} \left[\frac{1}{\lambda^*} + T - \tau \right] &= \frac{\beta}{a - \varepsilon} E_0 e^{\varepsilon T}. \end{aligned} \quad (40)$$

These terminal conditions may give the values of T and τ . And if we can compute those values which satisfy (40), the optimal time pattern of the rate of growth of investment is specified by (36) together with the values of T and τ .

which are determined by (40).

4. Existence of the Optimal Time Path

Optimal time pattern of the rate of growth of investment is determined by (36) and (40). However, the equation (40) does not have always the solutions. If the solutions do not exist, there does not exist the optimal time pattern of the rate of growth of investment.

The values of T and τ may satisfy

$$\tau = \frac{\varepsilon}{\lambda^*} T + \frac{1}{\lambda^*} \log \frac{\varepsilon}{a - \varepsilon} \frac{E_0}{I_0}, \quad (41)$$

$$\begin{aligned} \frac{a - \varepsilon}{\beta} \left[\frac{V_0}{E_0} - \frac{\beta}{\lambda^*} \frac{I_0}{E_0} \right] e^{-\varepsilon T} \\ = -\varepsilon \left(1 - \frac{\varepsilon}{\lambda^*} \right) T + \left[\left(1 - \frac{\varepsilon}{\lambda^*} \right) + \frac{\varepsilon}{\lambda^*} \log \frac{\varepsilon}{a - \varepsilon} \frac{E_0}{I_0} \right]. \end{aligned} \quad (42)$$

In this equation (42), the existence conditions of the solution T are

$$\begin{aligned} \frac{a - \varepsilon}{\beta} \left[\frac{V_0}{E_0} - \frac{\beta}{\lambda^*} \frac{I_0}{E_0} \right] e^{-\varepsilon T^*} \\ \leq -\varepsilon \left(1 - \frac{\varepsilon}{\lambda^*} \right) T^* + \left[\left(1 - \frac{\varepsilon}{\lambda^*} \right) + \frac{\varepsilon}{\lambda^*} \log \frac{\varepsilon}{a - \varepsilon} \frac{E_0}{I_0} \right], \end{aligned} \quad (43)$$

$$\lambda^* > \varepsilon, \quad (44)$$

where T^* satisfies

$$\frac{a - \varepsilon}{\beta} \left[\frac{V_0}{E_0} - \frac{\beta}{\lambda^*} \frac{I_0}{E_0} \right] e^{-\varepsilon T^*} = 1 - \frac{\varepsilon}{\lambda^*}.$$

Analyzing these relations, we have the following characteristics: (i) The larger λ^* generates the smaller T . (ii) The conditions (43) cannot be satisfied by larger λ^* than certain level λ^{**} . These characteristics suggest that the optimal time pattern of the rate of growth of investment can be determined by $[T^{**}, \tau^{**}]$ corresponding to λ^{**} .

The numerical values of our programming model give the following equations,

$$\begin{aligned} \tau &= \frac{1}{\lambda^*} (0.07T + 0.39), \\ \left(1.278 - \frac{0.047}{\lambda^*} \right) e^{-0.07T} &= \left(\frac{0.005}{\lambda^*} - 0.07 \right) T + 1 - \frac{0.043}{\lambda^*}. \end{aligned}$$

Analyzing these equations, we have the following results

λ^*	T^*	Solution	τ^{**}	T^{**}
0.075	32.0	exist		
0.08	24.0	exist		
0.09	17.0	exist		
0.095	15.6	exist		
$\lambda^{**} = 0.096$	15.3	exist	16.5	17.0
0.097		not exist		

Thus we have the optimal time pattern of the rate of growth of investment.

$$\begin{aligned} \lambda(t) &= 0.096 & \text{for} & \quad 0 \leq t \leq 16.5, \\ \lambda(t) &= 0 & \text{for} & \quad 16.5 \leq t \leq 17.0. \end{aligned} \quad (45)$$

Now let us remember that we have postulated that the upper limit of the rate of growth of investment, which is permitted by absorptive capacity for additional investment, is 0.15 in our programming model. However, in our numerical example, the maximum rate of growth of investment is 0.096. This value is far below the upper limit. This is because the terminal situation of our programming model is connected with the balanced growth after completing the taking-off period and the economy has reached the self-sustained situation. Otherwise, we could take the maximum rate of growth of investment at the upper limit (0.15). For example, when the objective of our programming model is to maximize an accumulated consumption over certain time horizon, we could take the maximum rate of growth of investment at 0.15. Thus, in the problem to minimize the required time for taking-off a developing economy into the self-sustained and balanced growth situation, there is a possibility to take the maximum rate of growth of investment below the upper limit permitted by the absorptive capacity for additional investment.

III. ALTERNATIVE STRATEGIES FOR INDIA

The Indian Planning Commission has made "the long-term perspective" as a basis for its Fourth Five Year Plan 1969-74 (Draft).⁷ In this perspective the net capital inflow from abroad is postulated to decline steadily and approach zero by the end of the Fifth Five Year Plan (1978-79) (see Table III). In the draft, we can read little reason for this assumption apart from the desire to become independent of foreign assistance. The draft does not consider the self-sustained and balanced growth situation after 1978-79.

In order to examine the effects of alternative strategies for Indian economy, we start from the basic situation which is composed of both the base year data (see Table I) described by the Indian long-term perspective and the structural

TABLE I
BASE YEAR DATA: 1967-68
(Rs. 100 crores at 1967-68 prices)

I_0	net investment	32.5
V_0	net domestic product	306.7
S_0	savings	22.0
F_0	net capital inflow	10.5
E_0	exports	12.0
M_0	imports	15.4
C_0	consumption	284.7

Source: Government of India, *Fourth Five Year Plan 1969-74 (Draft)*, 1969.

⁷ See [7].

parameters (see Table II) modified by the Indian plan values. The base year data depends on the actual situation of Indian economy at 1967-68, and the data are expressed in terms of Rs. 100 crores at 1967-68 prices. And, to determine plausible values for the structural parameters in our basic model and possible alternative variations in them, we use the plan documents and other analysis of the Indian economy.⁸ However, we do not design our results as a critique of the Indian plan but suggest some possibilities to make more effective development strategies.

TABLE II
VALUES OF STRUCTURAL PARAMETERS

	Basic Solution	Indian Plan		
		1967-74	1974-79	1979-81
λ^* : maximum rate of growth of investment, in %.	9.6	10.0	10.0	—
$\bar{\lambda}$: upper limit of rate of growth of investment, in %.	15.0	—	—	—
β : investment productivity.	0.35	0.50	0.42	0.42
s : marginal rate of savings.	0.28	0.28	0.28	0.28
ϵ : rate of growth of exports, in %.	7.0	7.0	7.0	7.0
m : import ratio.	0.050	0.048	0.045	0.043
n : rate of growth of population, in %.	2.5	2.5	—	1.7

Source: Same as Table I.

In the following arguments, we first examine the optimal growth path corresponding to the basic solution. The objective of optimization in our model is designed to minimize the required time for taking-off the Indian economy into the self-sustained and balanced growth situation. And, next, we examine the effects of alternative strategies on Indian economy. For the examinations, we postulate four alternative strategies: (1) to increase the initial level of net capital inflow from abroad [A-1], (2) to increase the investment productivity [A-2], (3) to decline the import ratio after completing the take-off process [A-3], and (4) to raise the rate of growth of exports [B-1]. And also, we make three combinations of these alternative strategies: (5) to combine (1) and (3) [A-13], (6) to combine (2) and (3) [A-23], and (7) to combine (1), (2) and (3) [A-123]. To take together with these alternative strategies, we can get some ideas of the interrelations among these alternative strategies and of the effective combinations of some alternative strategies for development.

1. The Basic Solution for India

The model of our analysis has been constructed as follows: to find the time pattern of the rate of growth of investment (26) to minimize the required time (27) for taking-off a developing economy into the self-sustained and balanced growth situation subject to the constraints (28). And, applying the numerical values of the base year data (Table I) and the structural parameters (Table II), we

⁸ See [5, 6, 7], [10], and [8].

get a set of solutions, which will be called *the basic solution*.

The growth of NDP in the basic solution is shown in Figure 1 and Tables III and IV. This growth path is closed by the NDP growth path in Indian plan. The time patterns of the other variables in the basic solution are shown in Figures 2 and 3 and Table IV. The basic solution does not explicitly consider a progress of import substitution. However, implicitly, we can consider it. For example, we can expect a possibility that the fraction of investment goods in total imports to decline as a result of the progress of import substitution.

The Basic Pattern of Investment and Domestic Savings: The basic solution shows that the developing process can be divided into the following three phases:

Phase	Description	Period
I	Maximum investment growth	1967-83
II	Adjustment to self-sustained and balanced growth situation	1984-85
III	Self-sustained and balanced growth	after 1985

In Phase I, investment grows at the maximum rate permitted by conditions for

Fig. 1. Growth of NDP in Indian Long-term Perspective and the Basic Solution to Model

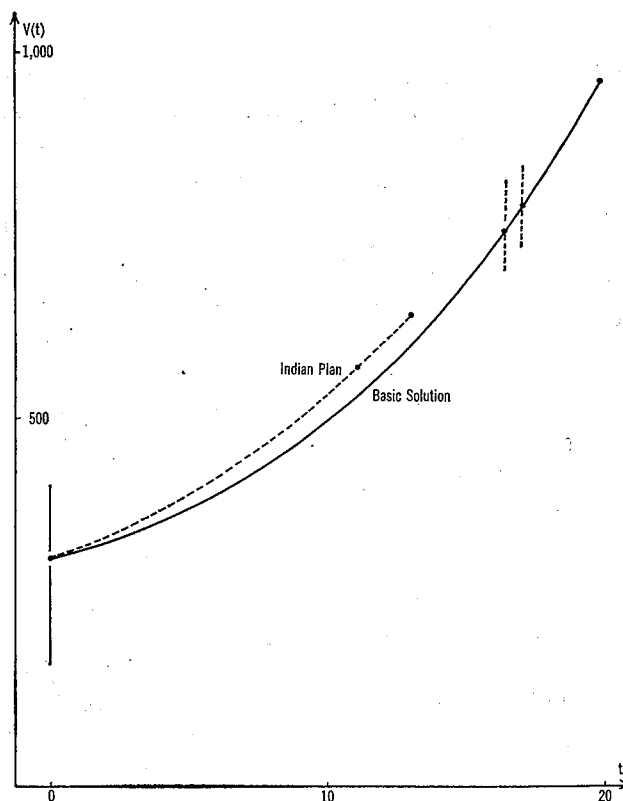


TABLE III
GROWTH RATES AND SIGNIFICANT RATIOS FOR THE BASIC SOLUTION OF
THE MODEL AND THE INDIAN PLAN

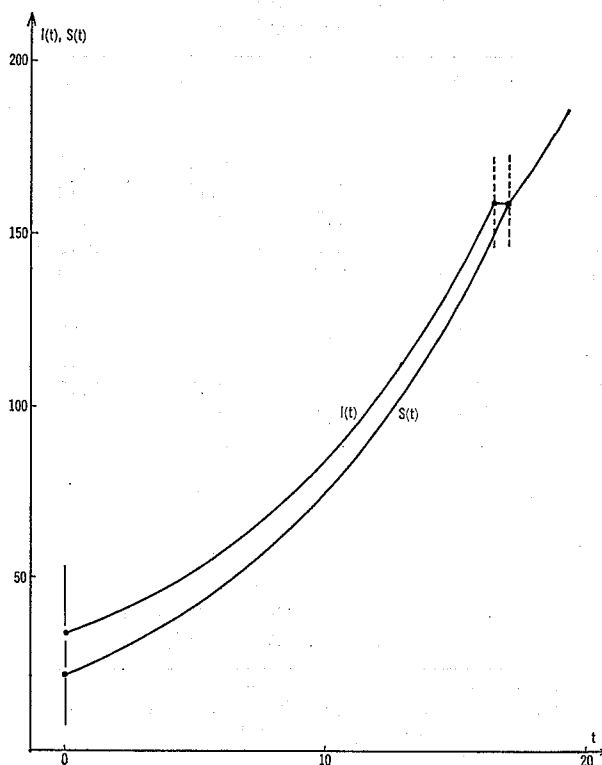
	Rate of Growth in %	V_n/V_0	I_n/V_n	S_n/V_n	F_n/V_n
Basic Solution					
1967-68	3.6	1.0	0.11	0.07	0.04
1973-74	5.0	1.3	0.15	0.12	0.03
1978-79	6.2	1.7	0.18	0.16	0.02
1980-81	6.7	2.0	0.19	0.18	0.01
1985-86	7.0	2.8	0.20	0.20	0.00
Indian Plan					
1967-68	4.0	1.0	0.11	0.07	0.04
1973-74	5.4	1.4	0.14	0.13	0.01
1978-79	6.6	1.8	0.17	0.17	0
1980-81	7.0	2.1	0.18	0.18	0
1985-86	—	—	—	—	—

Source: Same as Table I.

TABLE IV
VARIABLE VALUES IN THE BASIC SOLUTION
(Rs. 100 crores at 1967-68 prices)

Time	Plan Year	Invest- ment (I)	Net National Product (V)	Savings (S)	Net Capital Inflow (F)	Con- sumption (C)	Exports (E)	Imports (M*)	Traditi- onal Imports (M)
0	1967-68	32.5	306.7	22.0	10.5	284.7	12.0	22.5	15.4
1	1968-69	35.6	318.6	25.2	10.4	293.2	12.9	23.3	15.9
2	1969-70	39.4	331.9	29.1	10.3	302.9	13.8	24.1	16.6
3	1970-71	43.4	346.3	33.2	10.2	313.2	14.8	25.1	17.3
4	1971-72	47.8	362.2	37.7	10.1	324.6	15.9	26.0	18.1
5	1972-73	52.5	379.7	42.5	10.0	337.2	17.0	27.1	19.0
6	1973-74	57.8	398.9	47.9	9.9	351.0	18.3	28.2	19.9
7	1974-75	63.6	420.2	53.8	9.8	366.4	19.6	29.4	21.0
8	1975-76	70.0	443.6	60.4	9.6	383.2	21.0	30.6	22.2
9	1976-77	77.1	469.4	67.7	9.4	401.2	22.5	32.0	23.5
10	1977-78	84.9	497.7	75.7	9.2	422.1	24.2	33.4	24.9
11	1978-79	93.4	528.9	84.4	9.0	444.5	25.9	35.0	26.4
12	1979-80	102.8	563.1	94.0	8.8	469.2	27.8	36.7	28.2
13	1980-81	113.2	601.1	104.6	8.6	496.4	29.8	38.4	30.1
14	1981-82	124.7	642.9	116.4	8.3	526.5	32.0	40.3	32.1
15	1982-83	137.2	688.4	129.1	8.0	559.3	34.3	42.3	34.4
16	1983-84	150.9	738.4	143.2	7.7	595.2	36.8	44.6	36.9
	middle of								
16.5	1983-84	158.9	766.5	151.5	7.4	615.2	38.3	45.8	38.3
17	1984-85	158.9	794.2	158.9	0	635.7	39.7	39.7	39.7
18	1985-86	170.3	852.2	170.3	0	682.3	42.6	42.6	42.6
19	1986-87	182.7	913.4	182.7	0	731.0	45.7	45.7	45.7
20	1987-88	196.0	980.1	196.0	0	784.4	49.0	49.0	49.0

Fig. 2. Savings and Investment in the Basic Solution



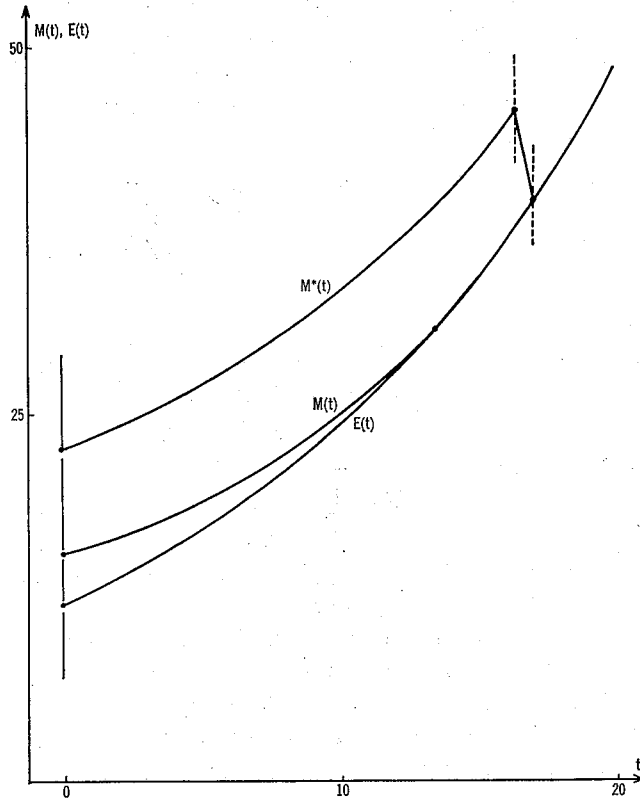
taking-off the economy into the self-sustained and balanced growth situation in minimum time horizon. However investment may not always rise more rapidly than domestic savings, there exists "saving-gap" (shortage of investment over domestic savings) through Phase I. And this saving-gap is financed by the net capital inflow from abroad.

When the growing investment reaches a certain level that can assure the balanced growth at a specified rate after some time (at the end of Phase I), the economy shifts to the Phase II. The basic solution shows that the turning time point comes at the middle of 1983-84.

In Phase II, investment ceases to grow and the economy adjusts itself to the self-sustained and balanced growth situation. On the process, while investment is kept at the certain level that is reached at the end of Phase I, domestic savings continues to increase because NDP is still increasing even for the constant level of investment. Thus the saving-gap decreases through Phase II. And Phase II is closed at the same time as the saving-gap is closed. This also means that the net capital inflow from abroad decreases rapidly and becomes zero at the end of Phase II. Then the economy reaches the self-sustained situation, when the take-off process is completed.

At the end of Phase II, ratio of investment to NDP becomes a specified level

Fig. 3. Exports and Imports in the Basic Solution



that assures an appointed rate of balanced growth for a given investment productivity.

Phase III starts in the year when the economy is adjusted to the self-sustained and balanced growth situation. And, in Phase III, the self-sustained and balanced growth at the specified rate is maintained.

The Basic Pattern of NDP: Time pattern of NDP growth is different corresponding to the respective three phases mentioned above. In Phase I, the rate of growth of NDP is accelerated because the rate of growth of investment is kept at constant level which is higher than the rate of growth of NDP. And, at the end of Phase I, the rate of growth of NDP reaches to 7.7 per cent level, which is higher than the balanced growth rate that will be expected in Phase III.

In Phase II, which is the adjustment process of the economy to the self-sustained and balanced growth situation, the level of investment is kept constant, while the NDP increases. Then the investment ratio decreases through Phase II. Thus, for the constant investment productivity, the rate of growth of NDP decreases. This reaches 7.0 per cent (the balanced rate of growth) at the end of Phase II. And, in Phase III, NDP grows at the balanced rate.

The Basic Pattern of Net Capital Inflow: Motive power of economic growth in

the take-off process is a growth of investment. Shortage of the investment over domestic savings (the saving-gap) is financed by net capital inflow from abroad. We postulate that the net capital inflow is treated as a part of economic aid from the developed countries. As well known in the *ex-post* balance equation in aggregate terms, the saving-gap is filled by the trade-gap (imports *minus* exports),

$$I - S = M^* - E.$$

This relation is true in the *ex-post* dimension. However, in the *ex-ante* dimension, the relation is not always true between the saving-gap and the trade-gap. Actually, the *ex-ante* saving-gap may be larger than the *ex-ante* trade-gap in developing countries,

$$I - S > M - E,$$

where M is structurally required imports which we call "the traditional imports." While we refer to M^* as adjusted imports for the *ex-post* balance equation. According to the above-mentioned inequality, M^* may be larger than M ,

$$M^* - M > 0.$$

This difference is required for maintaining investment at the scheduled level. Thus the net capital inflow is always equal to the saving-gap or the *ex-post* trade-gap.

Actually, there exists a positive saving-gap on the take-off process in a developing economy. Whether the saving-gap increases or decreases in Phase I on the take-off process depends on difference between the rate of growth of investment (λ) and the rate of growth of domestic savings ($s\beta$). If $\lambda > s\beta$, the saving-gap increases. And if $\lambda < s\beta$, this decreases. However, in Phase II the saving-gap inevitably decreases and becomes zero at the end of Phase II. And, in Phase III which is the self-sustained situation, there does not exist the saving-gap.

2. Effects of Alternative Strategies

Now we are in a position to examine some effects of alternative strategies. As mentioned above, we considered four alternative strategies and three combined strategies of them. The numerical values of base year data and structural parameters in these respective strategies are shown in Table V. And the effects of these respective strategies are also shown in Table V. Considering these alternative strategies and their combinations together, we will get some ideas of the interrelations among these alternative strategies and of the effective combinations of some alternative strategies for development.

The effect of increase in the initial level of net capital inflow from abroad in *ceteris paribus* condition of the basic solution [A-1] is mainly to increase the total requirement of net capital inflow through take-off process, while this strategy improves the growth pattern of consumption. The objective in our model is to minimize the time required for taking-off a developing economy into the self-sustained and balanced growth situation. Then the increase in the initial level of net capital inflow in *ceteris paribus* condition causes the maximum rate of growth of investment in Phase I of take-off process to depress. And, at the same time, this prolongs the required time for take-off process. While this strategy makes

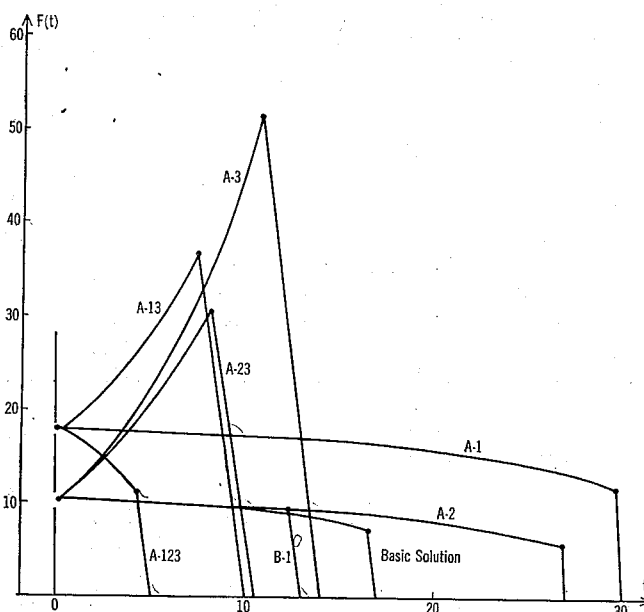
TABLE V
EFFECTS OF ALTERNATIVE STRATEGIES
(Rs. 100 crores at 1967-68 prices)

	B.S.	A-1	A-2	A-3	B-1	A-13	A-23	A-123
I_0	32.5					40.0		40.0
β	0.35		0.42				0.42	0.42
α	0.20		0.17		0.22		0.17	0.17
ϵ (%)	7.0				7.7			
m^*	0.05			0.04		0.04	0.04	0.04
λ^* (%)	9.6	7.7	7.8	15.0	12.0	15.0	15.0	14.0
τ	16.5	29.7	26.9	10.7	12.3	7.4	7.8	4.2
T	17.0	30.0	27.0	14.0	13.0	10.0	10.5	5.0
I_0	32.5	40.0	32.5	32.5	32.5	40.0	32.5	40.0
						121.4	104.7	72.0
I_{10}	84.9	86.4	70.9	145.7	107.9	121.4	104.7	102.5
	158.9			160.6	145.0			
I_{20}	196.0	186.6	154.9	244.4	248.7	244.5	203.5	206.3
		391.1	265.2					
I_{30}	394.1	391.1	327.3	492.2	537.2	492.3	410.0	415.5
V_0	306.7	306.7	306.7	306.7	306.7	306.7	306.7	306.7
						496.5	508.9	402.7
						607.0		430.9
V_{10}	497.7	517.6	513.4	570.7	526.6	607.0	605.6	611.4
	766.5			610.3	626.2		627.6	
	794.2			798.1	661.0			
V_{20}	980.1	973.1	964.5	1,214.7	1,133.6	1,222.5	1,220.1	1,231.5
		1,902.5	1,559.7					
		1,956.0	1,950.8					
V_{30}	1,970.0	1,956.0	1,963.0	2,546.2	2,449.0	2,461.4	2,457.7	2,479.8
s	0.28	0.22	0.19	0.28	0.35	0.33	0.26	0.40
\hat{C} (%)	4.7	5.7	5.6	5.3	4.4	5.0	5.5	4.4
\hat{c} (%)	2.2	3.2	3.1	2.8	1.9	2.5	3.0	1.9
$s\beta$ (%)	9.9	7.9	8.0	9.8	12.2	11.6	10.9	16.8
$\lambda^* - s\beta$	-0.3	-0.2	-0.2	5.2	-0.2	3.4	4.1	-2.8
ΣF	163.9	494.7	242.9	421.9	138.8	280.6	223.1	86.9

the marginal rate of savings lower than that in the basic solution. This lower rate, together with the constant investment productivity, may increase the rate of growth of net capital inflow even in relation to the decrease rate of growth of investment. These results are combined to increase the total requirement of net capital inflow through take-off process. In spite of the effects, the growth pattern of NDP is left unchanged. Even though the maximum rate of growth of investment in Phase I is decreased, the increased initial level of net capital inflow holds the long-term growth of investment to a similar pattern as the basic solution. This brings about the unchanged growth pattern of NDP.

Increase in the investment productivity [A-2], in *ceteris paribus* condition, may have the same effects as the effects of increase in the initial level of net capital inflow. This trends (1) to depress the maximum rate of growth of investment in Phase I and (2) to prolong the required time for take-off process. Simultaneously,

Fig. 4. Time Patterns of Net Capital Inflow in the Basic Solution and Alternative Strategies



the marginal rate of savings is reduced. These results may be associated to the increase in the total requirement of net capital inflow through take-off process. However, the initial level of net capital inflow is left unchanged in this case. Then the total requirement is smaller than that in the case created by the increase in the initial level of net capital inflow. And the growth pattern of NDP is not improved in this case.

These two alternative strategies are not effective in generating the self-sustained and balanced growth situation. When these strategies are applied individually, the effects are (1) to prolong the required time for take-off process, (2) to increase the total requirement of net capital inflow, (3) to keep the growth pattern of NDP unchanged in the basic solution, and (4) to make the growth pattern of consumption favorable compared to the basic solution. These results suggest that either increase in aid or increase of productivity may not be desirable to be taken individually but, as will be explained later, should be utilized in some effective combinations with the other strategies.

As a strategy that is effectively combined to the above-mentioned alternative strategies, we consider the strategy to decrease import ratio (a fraction of imports in NDP) [A-3]. We refer to this strategy as "progress of import substitution." This strategy trends (1) to increase the maximum rate of growth of investment in Phase I, and (2) to shorten the required time for take-off process. In our numerical example, the maximum rate of growth of investment reaches the upper limit of the rate of growth of investment, 15.0 per cent. Otherwise, the maximum rate might be increased much higher than 15.0 per cent. At the same time, the marginal

rate of savings is left unchanged. For the unchanged marginal rate of savings, the increased maximum rate of growth of investment causes an increase in the rate of growth of net capital inflow in Phase I. Then the total requirement of net capital inflow is increased. But, the terminal time point comes earlier than in the basic solution. Then the increase in total requirement of net capital inflow becomes less than expected. These results suggest that the progress of import substitution is effective strategy in generating the self-sustained and balanced growth situation.

To increase the rate of growth of exports (export promotion) [B-1] is also effective strategy in generating the self-sustained and balanced growth situation. This strategy tends (1) to increase the maximum rate of growth of investment in Phase I and (2) to shorten the required time for take-off process. Considering our numerical example, the effect of this strategy to shorten the required time for take-off process seemed to be more effective than in the strategy [A-3], while the growth pattern of NDP is left unchanged in the strategy [A-3].

Now let us examine some possibilities to effectively combine the above-mentioned alternative strategies. As explained above, the strategies [A-1] and [A-2] are not effective in generating the self-sustained and balanced growth situation, but the strategy [A-3] is effective for the same purpose. So we consider some possible combinations of ineffective strategy or strategies and effective strategy.

At first, let us consider the progress of import substitution accompanied with an increase in aid [A-13]. This strategy tends (1) to increase the maximum rate of growth of investment in Phase I and (2) to shorten the required time for take-off process. In this case, the maximum rate of growth of investment reaches the upper limit of the rate of growth of investment. And the terminal time point comes much earlier than in the basic solution. Moreover, in this case, the total requirement of net capital inflow is decreased and the growth pattern of NDP becomes favorable. These suggest that, however ineffective the strategy may be, if it is combined with other effective strategy, the combined effects may bring about more favorable results than that generated by the effective strategy individually.

To progress the import substitution along with the increase of investment productivity [A-23] has the same effects as the strategy [A-13] generates. Thus, we reach the same suggestions as the strategy [A-13] gives us.

Lastly, we examine the strategy in which the import substitution is progressed together with the increase in the initial level of aid and the increase of investment productivity [A-123]. As far as we are concerned, this strategy is the most effective. Taking this strategy, the terminal time point takes place in the shortest time horizon. And the growth pattern of NDP is prevented from making it more unfavorable than the pattern in the strategies [A-13] and [A-23]. Thus, even though both the increase in aid and the increase of productivity are ineffective strategies individually, if the strategies are taken together with the progress of import substitution, the combined effects become remarkably effective.

One of the main principle of the development economics shows that aid (net capital inflow in this concern) may be much more effective in generating self-sustained growth if it is heavily concentrated in the early stages of take-off process

rather than spread out more evenly over a longer period. Our results also confirm this principle. As stated above, the most effective strategy is [A-123]. In this strategy, the net capital inflow is heavily concentrated in the early stages of take-off process. This is due to the negative high rate of growth of net capital inflow: $\lambda^* - s\beta = -0.028$. Thus the terminal time point of take-off process in this strategy comes at 5.0. Consequently, the total requirement of net capital inflow is very small, while the growth of NDP shows very favorable pattern. However, the rate of growth of per-capita consumption is very low, because, in this case, domestic savings must be rapidly increased within a short time horizon.

In the alternative strategies [A-1] and [A-2], the net capital inflow is spread out evenly over a long time horizon. In these cases, it takes longer time for terminating their take-off process. And the growth patterns of NDP are less favorable than that in the basic solution. And, in the alternative strategy [A-3], the rate of growth of net capital inflow is very high: the rate of growth of investment ceils at the upper limit. But, in this case, the net capital inflow concentrates at the later stages of Phase I. Therefore, in this strategy, the terminal time point does not come early. However, the combined strategies of these alternative strategies [A-1], [A-2] and [A-3] show themselves much more effective than the separate strategies individually.

These results suggest, in order to get effective development strategy, that we should not take the alternative strategies separately but should use them in the combined forms. For example, the increased aid should be used in combination with either the increase of productivity or the progress of import substitution.

IV. CONCLUSION

Although our arguments in the present study have not proceeded far enough to examine the generality of our results, several aspects of development strategy seem to be clarified by considering a wide variety of alternative strategies and their combinations. The first is that there is a possibility of keeping the maximum rate of growth of investment below the permissible upper limit by an absorptive capacity of additional investment in the economy. Existing development theory shows that, since an increasing supply of investment funds by external resources is more favorable to development, the investment growth along the upper limit of the rate of growth of investment is always desired for take-off process. Contrarily, our conclusion suggests that the investment growth along the upper limit of the rate of growth of investment may not always be desired for optimal take-off process. Because this sort of investment growth may not lead the economy to a balanced growth situation in post take-off process.

We can find an optimal take-off process for a set of conditions. And we consider the more effective strategy for optimal take-off process to be that the strategy can terminate the take-off process within the shorter time horizon without giving any unfavorable effect to the growth pattern of NDP. Then the second aspect is that a more efficient optimal take-off process can be constructed by some possible combinations of individual strategies (for example, increase in aid, in-

crease of productivity, progress of import substitution, etc.). When these individual strategies are used separately, the resulting effects may be ineffective for generating the self-sustained and balanced growth situation. However, the possible combinations of these strategies may create more effective strategy for development.

There is a strong indication that the optimal growth strategy requires net capital inflow to be heavily concentrated in early stages of take-off process. However, unfortunately, developing countries are now unable to optimistically expect a lot of external resources. Thus, the developing countries should exert all possible efforts to institute more effective strategy for development so that it would be possible to realize more results with less aid requirement. The implications of these conclusions need to be tested in more realistic models.

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