

Heterogeneous contests and less informative signals*

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Abstract

We consider rank-order contests with heterogeneous agents in which the principal is restricted to use a fair contest (or symmetric contest), focusing on the optimal accuracy of output signals. As opposed to the absolute performance evaluation, we show it is optimal for the principal to deliberately make the signals noisier according to the degree of heterogeneity. Some economic interpretations of controlling noise are discussed.

JEL Classification: J31, M51, J33, D82

1 Introduction

In the classical contract theory with absolute performance evaluation, it is optimal for a principal to use all the available information on the agents' actions in order to induce their efforts.¹ This implies that the principal intends to make output signals more precise. In contests where the principal uses relative performance evaluation, a seminal work by Lazear and Rosen (1981) reveals that when the agents are homogeneous, their effort levels decrease as output signals become less precise. In contrast to these previous works, we show that in contests where the agents are heterogeneous, making the signal noisier can enhance the agents' effort levels and the principal's payoff.

Compensation schemes such as rank-order contests are those in which the agents are evaluated and awarded prizes on the basis of their relative order, rather than on

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¹See Holmström (1979).

the absolute levels of their performance. Competitions for employment and promotion, admission to universities, and student grading usually take the form of rank-order contests. Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) explore the basic properties of such rank-order contests. Green and Stokey (1983) indicate that when agents face severe common uncertainty, optimal tournaments dominate the optimal absolute evaluation contracts.

Agent heterogeneity in rank-order contests has been considered since the study by Lazear and Rosen (1981). Bhattacharya and Guasch (1988) indicate that comparisons across "self-selected" ability types are helpful for attaining efficiency in contests where the agents are heterogeneous. Dixit (1987) considers the effect of precommitment in contests with two heterogeneous agents, while Hurley and Shogren (1998) argue the effect of one-sided information asymmetry in a Cournot-Nash contest. Fullerton and McAfee (1999) analyze a research contest with n heterogeneous agents.

In contests with heterogeneous agents, placing handicaps for a low type agent is sufficient to attain efficiency, as Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) indicate. However, the option of handicapping is available only when the types of the agents are *common knowledge* and *verifiable* to the third party. If one or both of these assumptions do not hold, handicapping is no longer an option.

Affirmative actions and preferential arrangements for disabled persons are based on certain verifiable, socially-recognized ability gaps. If handicapping is based on certain unverifiable ability gaps, then the principal could be accused of favoritism from the aspect of *fairness*. In educational examinations, for instance, if a professor provides handicaps to certain students, she may be criticized for indulging in favoritism or biased treatment of students.²

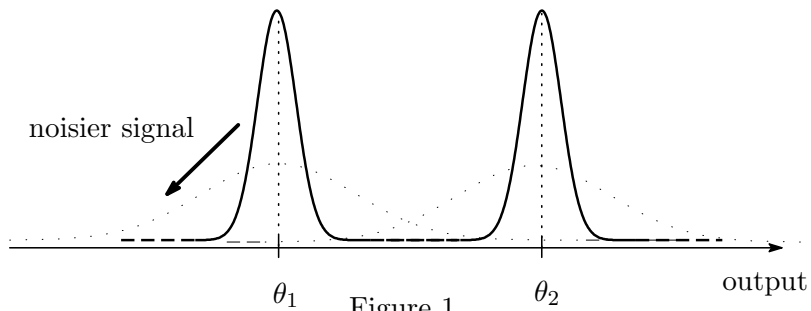
Handicapping is also unavailable if the agents' abilities are private information since the principal is required to identify each agent's ability. In this paper, we argue that in such cases, making the output signals noisier would be helpful in inducing agents' efforts and raising the principal's payoff.

There exists some literature on information manipulation in contests with homogeneous agents, but none that considers information manipulation in the case of heterogeneous agents. O'Keeffe et al. (1984) argue that in the case of the homogeneous agents, a principal may intentionally use noisier signals when the prize gap gets larger. If the prize gap is substantial, the agents would exert excessive effort, and imprecise signals would be required to reduce the agents' efforts to the optimal level. Nalebuff and Stiglitz (1983) also discuss the variance of noise in a contest with homogeneous agents, but they focus on the variance of *common shock*, not *individualistic* noise. In our model, the agents are heterogeneous and the principal can enhance the agents' efforts by using appropriately noisy signals.³

²If fairness is not a very important issue, like in a golf competition among co-workers, handicapping is adopted even if the ability gap is not verifiable.

³The possibility that the principal chooses a variance of the noise above the minimal level is not peculiar to contests. Focusing on renegotiation and commitment, Crémer (1995) indicates that more information does not necessarily make a better incentive scheme in individual contracts. This is because a principal with a precise monitoring that reveals the agent's type cannot commit to the threat of firing high quality agents with low performance levels.

The intuition of our argument is simple. A principal observes each agent’s output signals, consisting of the agent’s ability, effort, and individualistic noise and rewards only the agent submitting the highest signal. See the solid lines in Figure 1 that depict the distribution of each agent’s signal in the absence of effort, where θ_i , $i = 1, 2$ denotes each agent’s ability. Since there is a wide gap between θ_1 and θ_2 , there is little possibility of agent 1 winning even if he exercises some effort. This discourages agent 1 from working hard. Further, since agent 2 takes this into consideration, he feels confident winning and thus, lacks incentive to exercise effort.



However, by making the output signals noisier, the probability that agent 1 wins and agent 2 will lose increases. This is depicted by the dotted lines in Figure 1. Moreover, the prospect of luck induces agent 1 to work hard and not give up in order to increase the probability of such luck affecting his chance of winning the contest. This, in turn, drives agent 2 to work hard as well. Lazear (1998) argues that “the higher the noise, the lower the level of effort” (p.232) since noise reduces the value of effort. However, when the contestants are heterogeneous, higher noise can induce a higher level of effort. Clearly, if the signals are very noisy, the luck factor plays a larger role and making efforts becomes almost unnecessary, which is not in the principal’s interest. We calculate the optimal level of accuracy and clarify the relationship between the ability gap and accuracy.

Our result is relevant in certain realistic economic situations. For example, in educational examinations, it is difficult for a professor to handicap specific students unless they are recognized to be disabled or socially disadvantaged, despite her knowledge of each student’s ability. In such cases, she can encourage her students to study harder by making the output signals noisier, for example, by committing to present a narrower range of questions. Less able students may be able to obtain higher marks than more able students if they speculate on the questions and their suppositions are correct. This fact encourages the former to study harder, while inducing the latter to exert greater effort.

In section 2, we analyze the property of the optimal accuracy of noisy signals when handicapping is unavailable due to the unverifiability of the agents’ abilities. Here, we assume that the agents’ abilities are common knowledge. In section 3, we consider an example where the agents’ types are private information. We present the concluding remarks in Section 4.

2 Model and Analysis

2.1 Single Prize Contests

First, we consider single prize contests in which the principal controls the accuracy of the output signals as well as the value of the winning prize, W . There are two agents possessing different abilities. Each agent knows the other's ability and obtains zero utility if he does not participate in the contest. The principal knows the agents' abilities, however, she is restricted to designing a contest without handicapping, since the agents' abilities are not verifiable. The principal observes the output signals x_i of agent $i = 1, 2$, and awards the prize W only to the agent with the higher signal, while the other agent is not awarded anything. Each agent's signal x_i depends on i 's ability (type) θ_i , his effort level $e_i (\geq 0)$, independent idiosyncratic noise ε_i , and environmental uncertainty ξ , which is independent of ε_i .⁴ Specifically, we assume that

$$x_i = \theta_i + e_i + b\varepsilon_i + \xi, \quad (1)$$

where $b \in [\underline{b}, \infty)$ represents the accuracy of noisy signals that the principal can control⁵ and ε_i is independently drawn from the standard normal distribution $N[0, 1]$. Note that $Var(x_i) = b^2 + Var(\xi)$; thus, a smaller b corresponds to more accurate signals. The agents' effort levels are unobservable to the principal. Each agent has an identical cost function of effort $C(e)$ such that $C(0) = 0$, $C'(\cdot) > 0$, $C''(\cdot) < 0$, $C'(0) = 0$, and $C'(\infty) = \infty$.⁶ Agent i 's expected utility is

$$EU_i = P_i U(W) - C(e_i), \quad (2)$$

where P_i is the probability that agent i wins. $U(\cdot)$ satisfies $U'(\cdot) > 0$ and $U''(\cdot) < 0$. From (1), P_i is expressed as

$$\begin{aligned} P_i = Prob(x_i > x_j) &= Prob(\theta_i + e_i + b\varepsilon_i + \xi > \theta_j + e_j + b\varepsilon_j + \xi) \\ &= Prob\left(\frac{\theta_i + e_i - \theta_j - e_j}{b} > \varepsilon_j - \varepsilon_i\right) \\ &= Prob\left(\frac{\theta_i + e_i - \theta_j - e_j}{b} > \eta\right) \\ &= G\left(\frac{\theta_i + e_i - \theta_j - e_j}{b}\right), \end{aligned}$$

⁴Since contests are preferable to absolute evaluation contracts only when the environmental uncertainty is large, we assume that the agents face considerable environmental uncertainty.

⁵We restrict the lower bound of b in order to avoid a case in which the objective function is not concave in the relevant range and there is no pure strategy equilibrium. See footnote 2 of Lazear and Rosen (1981) and the Appendix of Nalebuff and Stiglitz (1983).

⁶In the existing literature such as Lazear and Rosen (1981), Bhattacharya and Guasch (1988), and Fullerton and McAfee (1999), low type agents are assumed to have higher marginal costs of effort, while we assume an identical cost function of effort for the purpose of simplicity. Our assumption, however, holds the property that low type agents have higher marginal costs of submitting any given expected signal levels.

where $\eta \equiv \varepsilon_j - \varepsilon_i$, $\eta \sim G(\eta)$; and, $G(\cdot)$ is the cdf of η . Note that η has the normal distribution $N[0, 2]$ since ε_i and ε_j are independent $N[0, 1]$ variables.

Each contestant chooses e_i to maximize (2). Assuming interior solutions, we can derive the first-order condition⁷

$$g\left(\frac{\theta_i + e_i - \theta_j - e_j}{b}\right) \frac{U(W)}{b} = C'(e_i), \quad i, j = 1, 2, i \neq j,$$

where $g(\cdot)$ is the pdf of η . Note that $g(\cdot)$ is a symmetric function. This implies that at the unique Nash equilibrium, each agent chooses the same effort level e^* such that⁸

$$e^* = C'^{-1}\left(g(\bar{\theta}/b) \frac{U(W)}{b}\right), \quad (3)$$

where $\bar{\theta} = |\theta_1 - \theta_2|$.⁹

We assume a risk-neutral principal whose payoff depends on the agents' *genuine output* $\sum_{i=1,2}(\theta_i + e_i)$. For simplicity, we assume that the principal bears no costs to change the level of b .¹⁰ The principal's payoff is expressed as

$$V = \theta_1 + e_1 + \theta_2 + e_2 - W.$$

The principal maximizes her payoff by setting W and b at the optimal level. The first order condition for b and (3) imply that¹¹

$$g'(\bar{\theta}/b^*) \left(-\frac{\bar{\theta}}{b^{*2}}\right) \frac{U(W)}{b^*} + g(\bar{\theta}/b^*) \left(-\frac{U(W)}{b^{*2}}\right) = 0. \quad (4)$$

Note that since $\eta \sim N[0, 2]$, we obtain $g'(\eta) = -\frac{1}{2}\eta g(\eta)$. Using this formula, (4) is reduced to

$$b^* = \bar{\theta}/\sqrt{2}. \quad (5)$$

This implies that as the gap between agents' abilities $\bar{\theta}$ increases, the principal uses coarser signals. Further, $e_1^* = e_2^*$ and (5) imply that at the equilibrium, the probability of agent i winning is invariant to the gap between the agents' abilities. The principal selects the accuracy level of output signals so that the high type agent wins with a

⁷We implicitly assume that the second order condition is always satisfied if $b > \underline{b}$.

⁸This property depends on the symmetry of $g(\cdot)$, identical cost functions, and the presence of only two agents. See Appendix for the uniqueness.

⁹If the principal can use an asymmetric (unfair) contest, her optimal action is to place the handicap of $x'_j = x_j + \bar{\theta}$ for the low type agent. If the agents are risk-neutral, handicapping can achieve the first-best outcomes in rank-order contests where the principal can control the winning prize and the losing prize.

¹⁰In the real world, it is indeed the case that the principal does not use precise signals because of their high costs. We ignore this not because we consider them less important, but in order to clarify our argument that the principal has an incentive to make the output signals deliberately noisy to induce her agents' efforts.

¹¹As noted previously, b is assumed to be above \underline{b} . If $\bar{\theta}$ is very small, the equilibrium b is \underline{b} , which is a corner solution, and equation (4) does not hold. In the following, we assume that $\bar{\theta}$ is sufficiently large to satisfy (4), except when we specifically refer to it.

probability of $G(\sqrt{2}) \approx 0.8413$ and the low type agent has a chance of winning with a probability of $1 - G(\sqrt{2})$.¹² Note that $G(\sqrt{2}) > 1 - G(\sqrt{2})$; hence, the high type agent is more likely to win than the low type agent.

Proposition 1 *In one-prize contests, the equilibrium level of b is proportional to the gap between the agents' abilities ($b^* = \bar{\theta}/\sqrt{2}$). The probability of agent i winning at the equilibrium is invariant to θ and only the order of θ_i matters.*

If the agents are identical, we can readily obtain $b = \underline{b}$ since we restrict the range of b in $[\underline{b}, \infty)$.

Corollary 1 *In the case of identical agents, the principal sets b at the lowest level.*

Next, we consider the optimal value of the winner's prize, W . For simplicity of calculation, we assume a quadratic cost function of effort, $C(e) = e^2/2$. From (3), the unique Nash equilibrium effort level is

$$e_1^* = e_2^* = e^* = g(\bar{\theta}/b) \frac{U(W)}{b}. \quad (6)$$

The first order condition for W and the property of normal distribution yield

$$2g(\bar{\theta}/b) \frac{U'(W^*)}{b} = 1$$

With these expressions, we obtain the winner's prize value and the agents' effort level:

$$W^* = U'^{-1} \left(\frac{\bar{\theta}}{2\sqrt{2}g(\sqrt{2})} \right), \quad \text{and} \quad (7)$$

$$e^* = \frac{\sqrt{2}g(\sqrt{2})}{\bar{\theta}} U \left(U'^{-1} \left(\frac{\bar{\theta}}{2\sqrt{2}g(\sqrt{2})} \right) \right). \quad (8)$$

Note that since $U(\cdot)$ is concave and $U'(\cdot)$ is a decreasing function, the inverse function of its derivative, U'^{-1} , is also a decreasing function. Therefore, e^* is decreasing in $\bar{\theta}$ as in the previous section and W^* is also decreasing in $\bar{\theta}$. This is because when $\bar{\theta}$ is large and the effort level of each agent is low, it does not pay to offer a big reward.

Further, we assume a CRRA utility function $U(W) = W^\alpha$, $\alpha \in (0, 1)$, for simplicity. The principal's payoff at the equilibrium is expressed as

$$V^* = (\theta_1 + \theta_2) + \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{2^{\frac{3}{2}} \alpha g(\sqrt{2})}{\bar{\theta}} \right)^{\frac{1}{1-\alpha}},$$

which is decreasing in the gap between the agents' abilities.¹³

Proposition 2 *The unique equilibrium effort level and the prize value are decreasing in $\bar{\theta}$. The principal's payoff also decreases as $\bar{\theta}$ rises when the agents' utility functions are given by the CRRA utility function $U(W) = W^\alpha$, $\alpha \in (0, 1)$.*

¹²Note that $\eta = \sqrt{2}$ corresponds to the point of inflection of $g(\eta)$ as $\eta \sim N[0, 2]$. This is derived from the property of normal distribution. (4) is rewritten as $g(\eta) + \eta g'(\eta) = 0$. The point that satisfies this equation corresponds exactly to the point of inflection when η is normally distributed.

¹³Note that we are considering the change in $\bar{\theta}$, while the sum of the agents' abilities $\theta_1 + \theta_2$ is constant.

2.2 Rank-Order Contests

Thus far, we have confined our models to a situation in which there is a single prize, i.e., only the winner is awarded a prize. However, a principal in general has the option to award a prize to the loser. The contest theory traditionally deals with such *rank-order contests*.¹⁴ The models discussed in the previous sections are specific in the sense that value of the losing prize is always zero.

If rank-order prize schemes are available, the agents' incentives to make efforts increase with the spread between the winning prize W_1 and the losing prize W_2 . Let ΔW denote this spread ($\equiv W_1 - W_2$) and ΔU represent $U(W_1) - U(W_2)$. The principal induces the agents to exert efforts up to the optimal level by adjusting the relative level of the prizes, ΔW (more precisely, ΔU) and selects the absolute level of each prize so as to extract the agent's surplus. Therefore, the individual rationality condition (IR) must be included in the constraints of the principal's maximization problem.

From this viewpoint, the distinction between single-prize contests, discussed in the previous section, and rank-order contests, discussed in this section, is whether the principal can "perfectly squeeze" the rent of the contestant who is *a priori* least likely to win, that is, his IR constraint is binding or not. This difference will relate mostly to whether or not it is possible for the principal to *punish* the loser ($W_2 < 0$). For, if the low type contestant strictly satisfies his IR constraint in the single-prize contest ($W_2 = 0$), then his IR constraint would never be binding in the rank-order contest insofar as $W_2 \geq 0$.¹⁵

In the rank-order contest, agent i solves the following problem.

$$Max_{e_i} \quad G\left(\frac{\theta_i - \theta_j + e_i - e_j}{b}\right) U(W_1) + \left[1 - G\left(\frac{\theta_i - \theta_j + e_i - e_j}{b}\right)\right] U(W_2) - C(e_i).$$

From the first-order conditions and the symmetry of $g(\cdot)$, we can derive

$$e_1 = e_2 = e^* = C'^{-1}\left(g(\bar{\theta}/b) \frac{\Delta U}{b}\right),$$

which is the same as those in the previous models except that $U(W)$ is replaced by ΔU .

We now address the principal's problem. Since the principal can extract the entire surplus of the low type agent by adjusting the levels of W_1 and W_2 , the IR constraint for him must be binding.¹⁶ Considering the fact that the probability of the low type agent winning at the equilibrium can be expressed by $G(-\bar{\theta}/b)$, the IR constraint is reduced to $G(-\bar{\theta}/b)U(W_1) + [1 - G(-\bar{\theta}/b)]U(W_2) = C(e^*)$. Formally, the principal solves the following maximization problem.

$$Max_{W_1, W_2, b} \quad \theta_1 + e_1 + \theta_2 + e_2 - (W_1 + W_2)$$

¹⁴The seminal work by Lazear and Rosen (1981) analyzes rank-order prize schemes. See also Nalebuff and Stiglitz (1983), Green and Stokey (1983), and Bhattacharya and Guasch (1988).

¹⁵Bhattacharya and Guasch (1988) refer to the constraint of $W_2 \geq 0$ as *limited liability*.

¹⁶Note that by the symmetry of the equilibrium effort level, each agent incurs the same costs of effort.

subject to

$$C'^{-1} \left(g(\bar{\theta}/b) \frac{\Delta U}{b} \right) = e_i = e^*, \quad i = 1, 2, \quad \text{and}$$

$$[1 - G(\bar{\theta}/b)] U(W_1) + G(\bar{\theta}/b) U(W_2) = C(e^*) \quad (\text{IR}).$$

An analytical solution to this problem is difficult. Unlike existing models where the principal chooses W_i , our model has another endogenous variable b affecting the agents' incentives to exercise effort, that in turn influences the optimal level of W_i through the IR constraint. This effect makes the first-order conditions complicated and difficult to solve, with the choice variable b in the cdf $G(\cdot)$ and the pdf $g(\cdot)$.

Therefore, we first solve the principal's problem with regard to W_i while b is given, and then analyze the effect of an increase in b .

To keep the model tractable, we assume that the agents are risk neutral,¹⁷ i.e., $U(W) = W$, and that they have a quadratic cost function $C(e) = e^2/2$. Thus, the unique Nash equilibrium effort level of both agents is $e_1 = e_2 = e^* = g(\bar{\theta}/b) \frac{\Delta W}{b}$.

These assumptions reduce the principal's problem to

$$\underset{W_1, W_2}{Max} \quad \theta_1 + \theta_2 + 2g(\bar{\theta}/b) \frac{\Delta W}{b} - (W_1 + W_2)$$

$$\text{s.t.} \quad [1 - G(\bar{\theta}/b)] W_1 + G(\bar{\theta}/b) W_2 = \frac{1}{2} \left[g(\bar{\theta}/b) \frac{\Delta W}{b} \right]^2.$$

By solving this problem, we obtain the following expressions;

$$\Delta W^* = B \frac{b}{g(\bar{\theta}/b)},$$

$$W_1^* = B \left[\frac{1}{2} B + \frac{bG(\bar{\theta}/b)}{g(\bar{\theta}/b)} \right], \quad \text{and}$$

$$W_2^* = B \left[\frac{1}{2} B - \frac{b(1 - G(\bar{\theta}/b))}{g(\bar{\theta}/b)} \right],$$

where

$$B = 1 + \frac{b(1 - 2G(\bar{\theta}/b))}{4g(\bar{\theta}/b)}.$$

Thus, the equilibrium effort level is $e^* = g(\bar{\theta}/b) \frac{\Delta W}{b} = B$. Using these results, we can easily obtain the following expression of the principal's payoff:

$$V = \theta_1 + \theta_2 + B^2.$$

Therefore, maximizing V with regard to b is equivalent to maximizing B , which is equal to the equilibrium effort level. Differentiating B with b yields the following

¹⁷Although the utility function is different from that in the previous section, the main thrust of the analysis is whether the IR constraint is binding or not. If we assume the risk neutral utility function in the previous section, the optimal level of b is equal to $\bar{\theta}/\sqrt{2}$, although the optimal level of W is indeterminate.

expression:

$$\frac{dB}{db} = \frac{g(\bar{\theta}/b) \left[2 - \frac{\bar{\theta}^2}{b^2} \right] [1 - 2G(\bar{\theta}/b)] + \frac{4\bar{\theta}(g(\bar{\theta}/b))^2}{b}}{(2g(\bar{\theta}/b))^2}.$$

Since the denominator is always positive, the sign of $\frac{dB}{db}$ depends on the numerator. Note that the last term of the numerator is positive and $g(\cdot) > 0$ everywhere. Further, the symmetry of $g(\cdot)$ implies that $1 - 2G(\bar{\theta}/b) < 0$. Therefore, if $2 - \frac{\bar{\theta}^2}{b^2} \leq 0$, or $b \leq \bar{\theta}/\sqrt{2}$, then $\frac{dB}{db}$ is always positive and the principal can increase her profit by raising b . Since the optimal b in the previous section without the IR constraint is $\bar{\theta}/\sqrt{2}$, the principal will use coarser signals in rank-order contests as compared with single prize contests. Note that this property arises from the imposition of the binding IR constraint. We summarize this result as a proposition.

Proposition 3 *The optimal level of b in rank-order contests is larger than that in single prize contests where the IR constraint is not binding.*

The intuition is as follows. If the principal is restricted to rewarding only the winner, the IR constraint is not binding and the principal's problem of choosing b is equivalent to maximizing the agents' effort levels. In contrast, when a prize to the loser is also available, the principal can extract the agents' surplus up to the level where the IR constraint becomes binding. We can easily check this by differentiating the IR constraint with respect to b and using the property of normal distribution that at $b = \bar{\theta}/\sqrt{2}$, an increase in b has the effect of relaxing the IR constraint. This means that in the model with unbinding IR constraints, the agents exert excessive efforts and curtail their own surplus. Therefore, in the case of rank-order contests where the IR constraint is binding, by raising b slightly above $\bar{\theta}/\sqrt{2}$ the principal can make a margin for cutting down the absolute value of W_1 and W_2 whilst keeping the spread ΔW constant.

2.3 Recruitment, Admission, and Research Contests

Here, we have considered those contests in which the principal profits from all the contestants. However, in the case of recruitment, admission to universities, and research contests,¹⁸ the principal is only concerned with the genuine output of the successful contestants. Note that the problem faced by the agents is the same as that in section 2.1, and if they have quadratic disutility function of effort, $C(e) = e^2/2$, their Nash equilibrium effort levels are described by (6). Therefore, the principal maximizes her expected payoff

$$EV = G\left(\frac{\theta_1 - \theta_2}{b}\right) \theta_1 + \left[1 - G\left(\frac{\theta_1 - \theta_2}{b}\right)\right] \theta_2 + e^* - W$$

subject to (6).

¹⁸Research contests are organized to promote research on specific projects and the sponsor employs the highest performing products or techniques. See Taylor (1995) and Fullerton and McAfee (1999).

The appendix shows that in this case, the optimal level of b is lower than those in the previous sections. This is quite intuitive. In this environment, the principal must choose the agent with the higher genuine output as well as extract this agent's effort. Using coarser signals entails a greater risk of choosing an inappropriate agent, although it might be helpful to extract both agents' efforts.

3 Incomplete Information

In this section, we consider single prize contests with incomplete information. We assume that the agent's type is private information and can take the value of θ^H or θ^L with a probability of $\frac{1}{2}$, respectively. Each agent has a quadratic cost function $C(e) = e^2/2$. The expected utility for each type of the agent $i = 1, 2$ is

$$\begin{aligned} EU_i^H &= \left[\frac{1}{2}G\left(\frac{e_i^H - e_j^H}{b}\right) + \frac{1}{2}G\left(\frac{\bar{\theta} + e_i^H - e_j^L}{b}\right) \right] U(W) - (e_i^H)^2 \\ EU_i^L &= \left[\frac{1}{2}G\left(\frac{e_i^L - e_j^L}{b}\right) + \frac{1}{2}G\left(\frac{\bar{\theta} + e_i^L - e_j^H}{b}\right) \right] U(W) - (e_i^L)^2. \end{aligned}$$

where e_i^H and e_i^L represent the effort levels of high type and low type agents $i = 1, 2$, respectively, and $\bar{\theta} \equiv |\theta^H - \theta^L|$.

The agent $i = 1, 2$ chooses his effort level to maximize his expected utility. The first-order condition for the high type agent is expressed as

$$\left[g\left(\frac{e_i^{H*} - e_j^H}{b}\right) + g\left(\frac{\bar{\theta} + e_i^{H*} - e_j^L}{b}\right) \right] \frac{U(W)}{2b} = e_i^{H*}.$$

In a symmetric equilibrium, $e_i^{H*} = e_j^{H*}$ and $e_i^{L*} = e_j^{L*}$. Thus, the first-order conditions for the high type agent and low type agent become

$$\begin{aligned} \left[g(0) + g\left(\frac{\bar{\theta} + \bar{e}^*}{b}\right) \right] \frac{U(W)}{2b} &= e^{H*}, \\ \left[g(0) + g\left(\frac{-\bar{\theta} - \bar{e}^*}{b}\right) \right] \frac{U(W)}{2b} &= e^{L*} \end{aligned}$$

respectively, where $\bar{e}^* \equiv e^{H*} - e^{L*}$. Since $g(\cdot)$ is a symmetric function, the left hand sides of the above-mentioned two equations are the same, and we obtain $e^{H*} = e^{L*} = e^*$ and $\bar{e}^* = 0$. Thus, the optimal effort level is

$$e^{H*} = e^{L*} = e^* = \left[g(0) + g\left(\frac{\bar{\theta}}{b}\right) \right] \frac{U(W)}{2b}.$$

Now, the principal chooses the value of W and b in order to maximize her expected payoff

$$\begin{aligned} V &= \frac{1}{4}(2\theta^H + 2e^*) + \frac{1}{2}(\theta^H + \theta^L + 2e^*) + \frac{1}{4}(2\theta^L + 2e^*) - W \\ &= \theta^H + \theta^L + 2e^* - W. \end{aligned}$$

The first-order condition for b is

$$g' \left(\frac{\bar{\theta}}{b} \right) \left(-\frac{\bar{\theta}}{b^2} \right) \frac{U(W)}{b} + \left[g(0) + g \left(\frac{\bar{\theta}}{b} \right) \right] \left(-\frac{U(W)}{b^2} \right) = 0.$$

By the property of $g(\cdot)$, this expression is reduced to

$$\left(\frac{\bar{\theta}^2}{2b^2} - 1 \right) g \left(\frac{\bar{\theta}}{b} \right) = g(0). \quad (9)$$

This implies that the optimal b is irrelevant with regard to W . Since $g(\cdot) > 0$, $\frac{\bar{\theta}^2}{2b^2} - 1$ must be positive, which implies $b < \bar{\theta}/\sqrt{2}$. The optimal b in the incomplete information case is smaller than that in contests with complete information. By differentiating (9) with respect to $\bar{\theta}$ and rearranging it, we can obtain

$$\frac{db}{d\bar{\theta}} = \frac{b(\bar{\theta})}{\bar{\theta}} > 0.$$

Therefore, the principal uses noisier signals when the ability gap between the agents increases, as in the complete information case. Further, the second derivative of $b(\bar{\theta})$ is

$$\frac{d^2b}{d\bar{\theta}^2} = \frac{b'(\bar{\theta})\bar{\theta} - b(\bar{\theta})}{\bar{\theta}^2} = 0,$$

which implies that the optimal $b(\bar{\theta})$ is linear in $\bar{\theta}$ and $\frac{b(\bar{\theta})}{\bar{\theta}}$ is constant.

Therefore, the probability that the high type agent wins $\frac{1}{2}G(0) + \frac{1}{2}G \left(\frac{\bar{\theta}}{b(\bar{\theta})} \right)$ is invariant to $\bar{\theta}$ since $\frac{b(\bar{\theta})}{\bar{\theta}}$ is constant. This is true for the low type agent as well.¹⁹

Proposition 4 *In an incomplete information setting, the optimal accuracy of the output signal b is linear and increasing in the ability gap $\bar{\theta}$. The probability of each agent winning at the symmetric equilibrium is invariant to $\bar{\theta}$.*

Next, we consider the optimal prize value. The first-order condition for W is

$$\left[g(0) + g \left(\frac{\bar{\theta}}{b} \right) \right] \frac{U'(W)}{b} = 1.$$

By differentiating the above equation with respect to $\bar{\theta}$, we can obtain

$$\frac{dW}{d\bar{\theta}} = \frac{1}{U''(W)} \left(-\frac{\frac{b}{\bar{\theta}}}{g(0) + g \left(\frac{\bar{\theta}}{b} \right)} \right) < 0$$

Using these results, we can derive

$$\frac{de^*}{d\bar{\theta}} = \left[g(0) + g \left(\frac{\bar{\theta}}{b} \right) \right] \left(\frac{U'(W)W'b - U(W)b'}{2b^2} \right) < 0.$$

Thus, the agents' efforts decrease as the ability gap increases.

¹⁹Although O'keeffe et al.(1984) do not analyze information manipulation in the case of heterogeneous agents and private information, they conjecture that "imprecise monitoring may be particularly helpful in motivating the most able workers". Our result that the principal intentionally uses noisier signals confirms their conjecture in a simple setting. I am indebted to an anonymous referee for this point.

Proposition 5 *The optimal prize level W and effort level e^* is decreasing in the ability gap $\bar{\theta}$.*

4 Concluding Remarks

In rank-order contests, using the most informative signals is not always the best strategy for a principal when there is a substantial gap between the agents' abilities. By making signals noisier and increasing the probability of the low type agent winning and the high type agent losing, the principal can induce both of them to work harder in order to increase her profit. The optimal level of the variance of the output signals is increasing in the gap between the agents' abilities.

Nalebuff and Stiglitz (1983) note that contests work best when all the participants are homogeneous. Generally, the more equal their ability, the greater is their effort. Making output signals noisier can be viewed as an indirect method to equalize their abilities.

Unlike handicapping, controlling the variance of output signals can be used when the ability gap is unverifiable since the contract is not directly dependent on this gap. This approach is also useful if the stakeholders are concerned about fairness since the contract is symmetrical for all agents.

There might be other means to increase the contestants' efforts in an environment similar to that described by us. One of the candidates will be "winning by a gap": the principal ranks one agent above the other only if that agent's signal is greater than his opponent's by a positive gap. Nalebuff and Stiglitz (1983) indicate that introducing a positive gap will improve the principal's payoff if the contestants are homogeneous and the distribution of idiosyncratic noise is symmetric. However, in the case of heterogeneous agents, the effect of introducing a gap on the principal's payoff is not clear and there is scope for further research.

An extension to n -agent models make the analysis more complicated, but it might be suggestive. In our model, allowing more than two agents breaks the symmetry of the Nash equilibrium effort level, which makes the analysis intractable. Usually, the greater the number of participants, the smaller are the ability gaps between the highest type and the second-highest type, between the second-highest type and the third-highest type, and so on. In such a case, the principal would employ more accurate signals since the average gap between the agents' types decreases.

We only analyze a simple model of incomplete information, where the agents' abilities can take either of two values, high or low, for tractability of the model. If an agent's ability is continuously and independently distributed, then the optimal effort level differs in accordance with his ability, which makes the model intractable. In general, high- and low-end types would exert more effort by worsening accuracy, but the middle-range types most likely reduce their efforts. The optimal accuracy level of the signals is dependent on the shape of the distribution of the agents' types. As this distribution becomes more concentrated around the mean, the expected aggregate effort level would decrease if the principal adopts less informative signals. Thus, the principal is likely to make the signals noisier only if she expects the ability gap to be considerable.

Appendix

The uniqueness of the Nash equilibrium

The first-order conditions for each agent are

$$\begin{aligned} g\left(\frac{\bar{\theta} + e_i - e_j}{b}\right) \frac{U(W)}{b} &= C'(e_i), \\ g\left(\frac{-\bar{\theta} + e_j - e_i}{b}\right) \frac{U(W)}{b} &= C'(e_j). \end{aligned}$$

Assume $e_i^* > e_j^*$. Since $C'(e_i^*) > C'(e_j^*)$, the first-order conditions imply $g\left(\frac{\bar{\theta} + e_i^* - e_j^*}{b}\right) > g\left(\frac{-\bar{\theta} - (e_i^* - e_j^*)}{b}\right)$. But the symmetry of $g(\cdot)$ implies that $g\left(\frac{\bar{\theta} + e_i^* - e_j^*}{b}\right) = g\left(\frac{-\bar{\theta} - (e_i^* - e_j^*)}{b}\right)$, which is a contradiction.

The same argument holds for $e_i^* < e_j^*$. Thus the equilibrium is proved to be unique.

Individual rationality in section 2.1

In section 2.1, we assume that the individual rationality constraints are satisfied and are not binding. By (3) and (5), the low type agent j 's utility function can be written as

$$EU_j^* = \left[(1 - G(\sqrt{2})) - \frac{\sqrt{2}g(2)}{\bar{\theta}} \right] U(W) \geq 0.$$

This condition is satisfied if $\bar{\theta} \geq \frac{\sqrt{2}g(2)}{1 - G(\sqrt{2})}$. Thus, if $\bar{\theta}$ is not very small, the agents' individual rationalities are satisfied and are not binding.

Optimal b in section 2.3

As in section 2.1, this problem can not be solved analytically. Thus, we use the same argument as in section 2.1. Assuming a CRRA utility function $U(W) = W^\alpha$, we obtain the following from the first-order condition with respect to W :

$$W = \left[\frac{\alpha g(\bar{\theta}/b)}{b} \right]^{\frac{1}{1-\alpha}}.$$

By substitution and arrangement, we can express the principal's expected payoff as

$$EV = G\left(\frac{\theta_1 - \theta_2}{b}\right) (\theta_1 - \theta_2) + \theta_2 + \alpha^{\frac{1}{1-\alpha}} (1 - \alpha) \left[\frac{g(\bar{\theta}/b)}{b} \right]^{\frac{1}{1-\alpha}}.$$

Differentiating EV with respect to b yields

$$\frac{dEV}{db} = -g(\bar{\theta}/b) \frac{(\theta_1 - \theta_2)^2}{b^2} + \alpha^{\frac{1}{1-\alpha}} \left[\frac{g(\bar{\theta}/b)}{b} \right]^{\frac{1}{1-\alpha}} \left[\frac{(\theta_1 - \theta_2)^2}{2b^4} - \frac{1}{b^2} \right].$$

When $b = \bar{\theta}/\sqrt{2}$, $\frac{dEV}{db} = -2g(\sqrt{2}) < 0$. This implies that the optimal b in this case is smaller than that in the previous sections.

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