SIZE OF ECONOMY AND AGGREGATE PRODUCTION FUNCTION
— The Case of Latin America —

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I. INTRODUCTION

THIS PAPER IS a presentation of an aggregate production function for the entire manufacturing sector: a variable-elasticity-of-substitution (VES) and a variable-return-to-scale (VRS) function according to the size of the economy.[1] The influence of the size of the economy upon the structure of production is one of the key issues in the discussions of development strategies and the effect of economic integration. It has not been treated explicitly in the form of aggregate production function. There are two ways of constructing this function, first by aggregation of production functions of subsectors, secondly by direct inspection of aggregate data. This paper is a presentation of a result induced by observation of the pooling data of the manufacturing sector in Latin American countries.

Section II presents results of the estimation of original and revised CES functions by data, and points out some basic features of the sizes of parameters. Section III presents an idea of VES and VRS function and its application to the data.

II. ESTIMATION OF CES FUNCTIONS

First we estimated the CES functions by the cross-section data of the manufacturing sector in Latin America. The necessary cross-section data classified by size of enterprises are available through the industrial census in six countries (Brazil—1959, Chile—1957, Colombia—1959, Costa Rica—1964, Peru—1963, and Mexico—1965) and four countries (except Costa Rica and Mexico) adopt similar classifications. The data of these four countries were regrouped into five groups by size (number of employees): Group I (5–9), II (10–19), III (20–49), IV (50–99), and V (100–). (See Appendix Table I).

[1] This paper is based on the joint discussion paper, Fukuchi-Hosono [4], written when the authors were working for the Economic Commission for Latin America. The contents present only personal views and the authors are fully responsible for possible errors or ommissions.

Table I shows the results of estimating the original version of the reduced form of CES function:

$$\left(\frac{V}{E}\right) = \gamma\left[\delta\left(\frac{K}{E}\right)^{-\rho} + (1-\delta)\left(\frac{L}{E}\right)^{-\rho}\right]^{-1/\rho} \tag{1}$$

(E, number of establishments in each size group; K/E, L/E, average number of capital stock or employees per establishment in each group)

by the original data and the regrouped data for each country and by the international pooling data (five groups; three or four countries).[2]

TABLE I

ESTIMATION OF CES FUNCTION

$\log(V/L)=a_0+a_1\log w$ $\log(V/K)=b_0+b_1\log r$

	Country	Number of Samples	$\hat{a}_0$ $(\hat{S}_{a_0})$	$\hat{a}_1$ $(\hat{S}_{a_1})$	$\hat{\bar{R}}$	$\hat{b}_0$ $(\hat{S}_{b_0})$	$\hat{b}_1$ $(\hat{S}_{b_0})$	$\hat{\bar{R}}$
1.	Brazil	5	1.2206 (0.0736)	0.8203 (0.1216)	0.9686	—	—	—
2.	Brazil	9	3.0593 (0.3848)	0.7302 (0.0610)	0.9765	−0.8047 (0.4861)	1.1556 (0.0689)	0.9878
3.	Colombia	5	1.0804 (0.0435)	1.2150 (0.1108)	0.9878	—	—	—
4.	Colombia	10	0.7905 (0.9875)	1.0413 (0.1471)	0.9287	0.4386 (0.0338)	0.9435 (0.1259)	0.9356
5.	Costa Rica	9	1.3370 (0.4678)	0.8468 (0.2888)	0.7425	0.4249 (0.0538)	0.9939 (0.0817)	0.9771
6.	Chile	5	1.3393 (0.0798)	1.1193 (0.1226)	0.9825	—	—	—
7.	Chile	8	−0.0820 (0.9381)	1.1658 (0.1153)	0.9719	0.0469 (0.7650)	1.0293 (0.0856)	0.9797
8.	Mexico	9	−1.0808 (0.0714)	0.3177 (0.0631)	0.8853	0.4469 (0.0230)	0.8463 (0.0327)	0.9948
9.	Peru	5	1.1365 (0.0443)	1.4546 (0.1169)	0.9904	—	—	—
10.	Peru	8	−5.8516 (0.8104)	1.5100 (0.0879)	0.9900	1.3867 (1.8601)	0.6426 (0.2013)	0.7932
11.	Co, Ch, Pe	5×3	1.1555 (0.0530)	1.0698 (0.1082)	0.9395	0.4241 (0.0222)	0.7799 (0.1044)	0.9006
12.	Br, Co, Ch, Pe	5×4	1.1738 (0.0525)	0.9752 (0.1007)	0.9159	0.4118 (0.0208)	0.7491 (0.0968)	0.8768

$$\log\left(\frac{V}{L}\right) = a_0 + a_1 \log w\,, \tag{2}$$

$$\log\left(\frac{V}{K}\right) = b_0 + b_1 \log r\,. \tag{3}$$

(V, added value; L, number of employees; K, value of capital; w, wage rate; r, average remuneration to capital.)

[2] The value of capital was not directly available cross-sectionally, except for horse power installed. This figure was converted to the value of capital by the multiplication of the national average ratio (value of capital-number of horse power installed).

We estimated the Cobb-Douglas-Durand (CDD) form without restriction but the results were not acceptable in all cases, because the coefficient of capital was so small, sometimes negative, while the sum of two coefficients showed a plausible scale effect. Table I shows that sometimes $\hat{a}_1$ diverges significantly from unity (for example, in Brazil and Peru) to create the misspecification error in the CDD estimation. There exists no clear difference in $\hat{a}_1$ and $\hat{b}_1$.[3]

Secondly we estimated the revised versions of CES by Diwan [3], and Dhrymes [2].[4]

$$\log\left(\frac{V}{L}\right) = c_0 + c_1 \log w + c_2 \log\left(\frac{L}{E}\right), \tag{4}$$

$$\log\left(\frac{V}{L}\right) = d_0 + d_1 \log w + d_2 \log\left(\frac{L}{E}\right) + d_3 \log\left(\frac{K}{L}\right). \tag{5}$$

We interpret (4) as the reduced form of the CES

$$\left(\frac{V}{E}\right) = \gamma\left[\delta\left(\frac{K}{E}\right)^{-\rho} + (1-\delta)\left(\frac{L}{E}\right)^{-\rho}\right]^{(-)n/\rho}, \tag{6}$$

and

$$\partial\left(\frac{V}{E}\right)\Big/\partial\left(\frac{L}{E}\right) = w. \tag{7}$$

Between (4) and (6) we have

$$c_0 = \left(\frac{n}{\rho+n}\right)\log\left(\frac{\gamma^{\rho/n}}{n(1-\delta)}\right), \quad c_1 = \frac{n}{\rho+n}, \quad c_2 = \frac{\rho(n-1)}{\rho+n}. \tag{8}$$

Or

$$\hat{n} = 1 + \frac{\hat{c}_2}{1-\hat{c}_1}, \quad \hat{\rho} = \frac{1+\hat{c}_2-\hat{c}_1}{\hat{c}_1}, \quad \hat{\sigma} = \frac{\hat{c}_2}{1+\hat{c}_2}. \tag{9}$$

We interpret (5) as the reduced form of the CES

$$\left(\frac{V}{E}\right) = \gamma\left[\delta\left(\frac{K}{E}\right)^{-\rho} + (1-\delta)\left(\frac{K}{L}\right)^{-m\rho}\left(\frac{L}{E}\right)^{-\rho}\right]^{(-)n/\rho}, \tag{10}$$

and

$$\partial\left(\frac{V}{E}\right)\Big/\partial\left(\frac{L}{E}\right) = w. \tag{11}$$

Between (5) and (10) we have

[3] In the past most empirical studies were done by interregional cross-section data (in nation or in the world), so this finding is not directly comparable with other observations concerning the clear difference between $\hat{a}_1$ and $\hat{b}_1$ (for example, Dhrymes [2]).

[4] Dhrymes [2] assumes the imperfect competition and replaces (7) by

$\partial(V/E)/\partial(L/E) = (1+\varepsilon)w/(1+\eta)$

so the interpretation is different.

$$d_0 = \left(\frac{-n}{n+\rho}\right) \log (1-\delta)(1-m)\gamma n,$$

$$d_1 = \frac{n}{n+\rho}, \quad d_2 = \frac{n(1+\rho)}{n+\rho}, \quad d_3 = \frac{m\rho n}{n+\rho}, \tag{12}$$

or

$$\hat{n} = 1 + \frac{\hat{d}_2}{1-\hat{d}_1}, \quad \hat{\rho} = \frac{1+\hat{d}_2-\hat{d}_1}{\hat{d}_1}, \quad \hat{\sigma} = \frac{\hat{d}_1}{1+\hat{d}_2}. \tag{13}$$

Table II shows the result of estimation of (4) with the estimated values of parameters ($\hat{n}$, $\hat{\rho}$, and $\hat{\sigma}$) by (9). The estimation of (5) showed the insignificance of a coefficient (mostly d_3) from zero in every country, so the results are not tabulated.[5]

In Table II we observe the clear tendency of decreased values of $\hat{\sigma}$ compared with Table I. In the international pooling case (case 11 and 12) $\hat{\sigma}$ was 1.07–0.98 in (2) and 0.78–0.75 in (3) but was 0.61–0.60 in (4), while $\hat{n}$ showed a significant scale effect 45–36 per cent in (4). We can deduct from this that in the manufacturing sector of developing countries in Latin America there exists a significant economy of scale effect and the a priori specification of the CES function without the scale effect (1) creates a positive misspecification error in the reduced-form estimation of $\hat{\sigma}$ by (2) or (3).[6] But on the other hand the estimation of (4) resulted in large divergences of $\hat{\sigma}$ between countries, suggesting that we are still neglecting the additional systematic variable.

Table II shows also an interesting tendency of $\hat{n}$ between countries. $\hat{n}$ is smallest in Brazil (1.10–1.07), the largest are in Colombia and Peru (1.58–1.30), and a middle in Chile (1.16), suggesting a hypothesis that $\hat{n}$ decreases according to the absolute size of the economy and to the per-capita level of income. This hypothesis can be supported by interregional observation within a country. Appendix Table III shows the basic data by four regions in Brazil. It is difficult to separate the effects of the per-capita income and population, but there exists a clear increasing tendency of the average size of establishment from the backward to the developed region. So we can infer that when the size of the economy, represented by population and per-capita income and their cross-effects, increases,

[5] Only in the cases of international pooling estimation were the results significant:

$$\log(V/L) = \underset{(0.1367)}{0.4940} + \underset{(0.1598)}{0.3953} \log w + \underset{(0.0348)}{0.1006} \log (L/E) + \underset{(0.1430)}{0.3414} \log (K/L)$$

$\hat{\bar{R}} = 0.9830$

(15 samples: Colombia, Chile, Peru.)

$$\log(V/L) = \underset{(0.1080)}{0.5786} + \underset{(0.1244)}{0.3339} \log w + \underset{(0.02615)}{0.06364} \log (L/E) + \underset{(0.1214)}{0.4505} \log (K/L)$$

$\hat{\bar{R}} = 0.9765$

(20 samples: three countries plus Brazil.)

So there remains a possibility to deepen the specification along the revised Diwan formula. But, of course, special reconsideration must be given to revise the data of capital (now we use the horse power instead of the value of capital).

[6] The value of $\hat{\sigma}$ (0.61–0.60) is near to 0.6947. The average of the 16 manufacturing subsectors by Arrow-Murata gave 0.8117–0.8292 as the averages of subsectors for 1953–59. (See Nerlove [7].)

TABLE II
CES FUNCTION
$\log (V/L) = a_0 + a_1 \log w + a_2 \log (L/E)$

	Country	Number of Samples	$(\hat{S}^{\hat{a}_0}_{a_0})$	$(\hat{S}^{\hat{a}_1}_{a_1})$	$(\hat{S}^{\hat{a}_2}_{a_2})$	$\hat{\bar{R}}$	$\hat{d}$	$\hat{n}$	$\hat{\rho}$	$\hat{\sigma}$
1.	Brazil	5	0.6760 (0.1964)	0.3132 (0.1910)	0.06642 (0.02344)	0.9938	3.344	1.0967	2.4048	0.2937
2.	Brazil	9	4.0859 (0.4734)	0.5149 (0.0918)	0.03728 (0.01393)	0.9893	2.903	1.0704	1.0191	0.4853
3.	Colombia	5	0.1068 (0.3630)	0.3583 (0.3249)	0.2343 (0.0872)	0.9974	2.729	1.3651	2.4449	0.2903
4.	Colombia	10	6.4102 (3.4731)	0.04688 (0.60260)	0.2856 (0.1693)	0.9433	1.819	1.2997	5.8064	0.1469
5.	Costa Rica	9	1.7316 (0.4924)	0.1421 (0.5170)	0.2214 (0.1400)	0.8266	1.970	1.2581	7.5955	0.1163
6.	Chile	5	−1.6067 (0.5536)	−1.4443 (0.4829)	0.3955 (0.0742)	0.9989	3.155	1.1619	—	—
7.	Chile	8	4.6344 (2.6950)	0.1117 (0.5700)	0.1455 (0.0772)	0.9837	—	1.1640	9.2550	0.0520
8.	Mexico	9	0.3701 (0.0040)	0.1133 (0.0142)	0.1723 (0.0093)	0.9981	2.204	1.1943	9.3467	0.0903
9.	Peru	5	−0.8042 (0.3004)	−0.3042 (0.2738)	0.4988 (0.0771)	0.9996	3.056	1.3825	—	—
10.	Peru	8	1.1250 (2.9116)	0.2739 (0.1106)	0.4219 (0.4444)	0.9935	1.075	1.5810	4.1912	0.1926
11.	Co, Ch, Pe	5×3	0.5449 (0.1593)	0.6846 (0.1229)	0.1412 (0.0359)	0.9740	—	1.4477	0.6670	0.5999
12.	Br, Co, Ch, Pe	5×4	0.6602 (0.1399)	0.6828 (0.1077)	0.1136 (0.0297)	0.9558	—	1.3581	0.6309	0.6132

the average size of establishment will increase enjoying an enlarged market and the key subsector can expect the development of highest scale effect. The development of the subsector will be followed up by the development of the related auxiliary subsectors, and the industrial structure will become more heavy-industry biased and investment-goods-oriented industry biased.[7]

So comparing the developed with the backward area we observe differences in the basic parameters: the decreasing tendency of scale parameter (n) and also the decreasing tendency of elasticity-of-substitution (σ). This suggests a variable-elasticity-of-substitution (VES) and variable-return-to-scale (VRS) production function.

$$\left(\frac{V}{E}\right) = \gamma\left[\delta\left(\frac{K}{E}\right)^{-\rho(Y,N)} + (1-\delta)\left(\frac{L}{E}\right)^{-\rho(Y,N)}\right]^{(-)n(Y,N)/\rho(Y,N)},$$

(Y, income; N, population.)

In the past the generalization of CES function to VES and to VRS were developed independently.[8] But the discussion above strongly suggests that the trends of basic parameters (n and σ) are related each other, so we had better avoid the independent generalization to protect ourselves from possible misspecification errors.

III. ESTIMATION OF A VES AND VRS FUNCTION

In the actual procedure we must estimate the following three functions simultaneously, approximating σ (or ρ) and n by some forms.

$$\left(\frac{V}{E}\right) = \gamma\left[\delta\left(\frac{K}{E}\right)^{-\rho} + (1-\delta)\left(\frac{L}{E}\right)^{-\rho}\right]^{-n/\rho}. \tag{14}$$

$$\rho = g(Y, N). \tag{15}$$

$$n = h(Y, N). \tag{16}$$

(14) can be replaced, under the assumption of perfect competition, to

$$\log\left(\frac{V}{L}\right) = c_0 + c_1 \log w + c_2 \log\left(\frac{L}{E}\right).$$

$$\hat{n} = 1 + \frac{\hat{c}_2}{1-\hat{c}_1}, \quad \hat{\rho} = \frac{\hat{c}_2 - \hat{c}_1 + 1}{\hat{c}_1}, \quad \hat{\sigma} = \frac{\hat{c}_1}{\hat{c}_2 + 1}.$$

7 Dhrymes [2] inferred that σ will take a relatively small figure in the investment-oriented industry in the United States.

8 For example, some trials on VES are done by Sato-Hoffman [9]. Concerning VRS see Soskice [10], Zellner-Revanker [11]. Of course we must recognize that the simultaneous recognition of VRS and VES generally creates another difficulty in the estimation procedure, which existed even in CES. See Maddala-Kadane [5], Bodkin-Klein [1], and Murata [6].

For the purpose of simultaneous estimation it is natural to assume c_1 and c_2 as functions of Y and N, so that $\hat{n}$ and $\hat{\rho}$ become functions of Y and N as postulated in (15) and (16). For the preliminary step we will assume that c_1 and c_2 are the linear functions of Y and N. This results in the next reduced form

$$\log\left(\frac{V}{L}\right) = A_0 + (A_1 + A_2Y + A_3N)\log w$$

$$+ (A_4 + A_5Y + A_6N)\log\left(\frac{L}{E}\right). \tag{17}$$

$$\therefore \quad \hat{n} = 1 + \frac{A_4 + A_5Y + A_6N}{(1 - A_1) - A_2Y - A_3N}$$

$$= \frac{(1 - A_1 + A_4) + (A_5 - A_2)Y + (A_6 - A_3)N}{(1 - A_1) - A_2Y - A_3N}. \tag{18}$$

$$\hat{\rho} = \frac{(1 - A_1 + A_4) + (A_5 - A_2)Y + (A_6 - A_3)N}{A_1 + A_2Y + A_3N}.$$

$$\hat{\sigma} = \frac{A_1 + A_2Y + A_3N}{(1 + A_4) + A_5Y + A_6N}. \tag{19}$$

We postulate

$$\frac{\partial\hat{n}}{\partial Y} < 0, \quad \frac{\partial\hat{n}}{\partial N} < 0, \quad \frac{\partial\hat{\sigma}}{\partial Y} < 0, \quad \frac{\partial\hat{\sigma}}{\partial N} < 0. \tag{20}$$

Data of the size variables are given in Appendix Table II.

The results are shown in Table III. Out of the four cases where the coefficients are significantly different from zero, the cases 7 and 9 give the negative estimates of $\hat{n}$ andin $\hat{\sigma}$ some cases:

	Case (7)		Case (9)	
	$\hat{n}$	$\hat{\sigma}$	$\hat{n}$	$\hat{\sigma}$
Brazil	—	—	1.1386	0.5410
Colombia	1.3222	negative	1.1972	1.0462
Chile	1.1698	negative	negative	0.6607
Peru	1.4199	0.0657	negative	1.0360

We will concentrate on cases 5 and 6:

	Case (5)		Case (6)	
	$\hat{n}$	$\hat{\sigma}$	$\hat{n}$	$\hat{\sigma}$
Brazil	1.0960	0.1459	1.1072	0.2141
Colombia	1.4191	0.5777	1.3952	0.5812
Chile	1.4419	0.5888	1.5119	0.6323
Peru	1.4750	0.6035	1.4503	0.6079

In case 5 we have

TABLE III
RESULTS OF VES VRS FUNCTION

$$\log(V/L)=A_0+A_1\log w+A_2Y\log w+A_3N\log w+A_4\log(L/E)+A_5Y\log(L/E)+A_6N\log(L/E)$$

	Country	Number of Samples	$(\hat{S}_{\hat{A}_0})$	$(\hat{S}_{\hat{A}_1})$	$(\hat{S}_{\hat{A}_2})$	$(\hat{S}_{\hat{A}_3})$	$(\hat{S}_{\hat{A}_4})$	$(\hat{S}_{\hat{A}_5})$	$(\hat{S}_{\hat{A}_6})$	$\hat{\bar{R}}$
1.	Co, Ch, Pe	5×3	0.4915 (0.1519)	1.6992 (0.8796)	−0.2295 (0.1906)		0.3091 (0.1071)	−0.03430 (0.02080)		0.9811
2.	Co, Ch, Pe	5×3	0.8054 (0.2130)	0.5060 (0.2402)		0.3821 (0.2028)	0.1108 (0.0390)		−0.02667 (0.03448)	0.9824
3.	Co, Ch, Pe	5×3	−0.3479 (0.2452)	1.2349 (0.4060)	−0.4287 (0.0795)	0.5845 (0.0875)	0.7711 (0.1270)	−0.1495 (0.0279)	0.2127 (0.0479)	0.9978
4.	Co, Ch, Pe	5×3	0.5449 (0.1593)	0.6846 (0.1229)			0.1412 (0.0359)			0.9740
5.	Br, Co, Ch, Pe	5×4	0.5302 (0.1374)	0.8427 (0.1136)	−0.03624 (0.01002)		0.1649 (0.0365)	−0.004445 (0.001734)		0.9767
6.	Br, Co, Ch, Pe	5×4	0.5634 (0.1393)	0.7803 (0.1111)		−0.08039 (0.02660)	0.1493 (0.0345)		−0.009806 (0.004484)	0.9728
7.	Br, Co, Ch, Pe	5×4	0.5066 (0.3004)	1.2751 (0.2980)	−0.2698 (0.0636)	0.5803 (0.1571)	0.1941 (0.1148)	−0.02374 (0.02587)	0.04953 (0.06008)	0.9887
8.	Br, Co, Ch, Pe	5×4	0.6602 (0.1399)	0.6828 (0.1077)			0.1136 (0.0297)			0.9558
9.	Br, Co, Ch, Pe	5×4	0.8652 (0.0934)	1.5946 (0.1845)	−0.2957 (0.0629)	0.6694 (0.1507)	0.05924 (0.02068)			0.9856

Note: Cases 4 and 8 are the results of estimation of (4), and are also tabulated for reference.

$$\hat{n} = \frac{0.3222 + 0.02779Y}{0.1573 + 0.03624Y}, \quad \frac{d\hat{n}}{dY} < 0\,. \tag{21}$$

$$\hat{\sigma} = \frac{0.8427 - 0.03624Y}{1.1649 - 0.00445Y}, \quad \frac{d\hat{\sigma}}{dY} < 0\,. \tag{22}$$

$\hat{\sigma}$ is defined for the value of Y

$$0 \leqq Y \leqq 23.25\,, \tag{23}$$

and less than 0.7234 (the value of $\hat{\sigma}$ when $Y = 0$).
In case 6 we have

$$\hat{n} = \frac{0.3690 + 0.07058N}{0.2197 + 0.08039N}, \quad \frac{d\hat{n}}{dN} < 0\,. \tag{24}$$

$$\hat{\sigma} = \frac{0.7803 - 0.08039N}{1.1493 - 0.00981N}, \quad \frac{d\hat{\sigma}}{dN} < 0\,. \tag{25}$$

$\hat{\sigma}$ is defined for the value of N

$$0 \leqq N \leqq 9.71\,, \tag{26}$$

and less than 0.6789 (the value of $\hat{\sigma}$ when $Y = 0$).
In cases 5 and 6 the original form of VES-VRS functions are as follows:

$$\left(\frac{V}{E}\right) = \gamma\left\{\delta\left(\frac{K}{E}\right)^{\left[\frac{0.0318Y+0.3222}{0.0362Y-0.8427}\right]} + (1-\delta)\left(\frac{L}{E}\right)^{\left[\frac{0.0318Y+0.3222}{0.0362Y-0.8427}\right]}\right\}^{\left(\frac{0.0362Y-0.8427}{0.0362Y+0.1573}\right)}, \tag{27}$$

$$\left(\frac{V}{E}\right) = \gamma\left\{\delta\left(\frac{K}{E}\right)^{\left[\frac{0.3690+0.0706N}{0.0804N-0.7803}\right]} + (1-\delta)\left(\frac{L}{E}\right)^{\left[\frac{0.3690+0.0706N}{0.0804N-0.7803}\right]}\right\}^{\left(\frac{0.0804N-0.7803}{0.0804N+0.2197}\right)}. \tag{28}$$

The results (27) and (28) with (21) (22) (24) (25) support our hypothesis (14), a VES-VRS function with the explicit size effect, although there exists intervals of possible definitions (23) (26) originating in the small number of samples and the necessary procedure of linear approximation.

The functions (27) or (28) can be applied to analyze the effects of economic integration on the production structure. The sum of the incomes of three countries (Colombia, Chile, and Peru) amounts to 13,876 (million dollars).
From (21) and (22) we get

$$\hat{n} = 1.1566, \qquad \hat{\sigma} = 0.3083\,. \tag{29}$$

This suggests that after the necessary transition period the industrial structure of integrated economy will be more investment-oriented-type biased and a significant scale effect will remain (16 per cent) in the long run. Of course, as mentioned earlier this type of aggregate analysis can be effectively supplemented by studies of subsectors and necessary aggregation in the future. (21), (22), (24), and (25) show that the two parameters (n and σ) decrease, that is, the economy of scale will diminish and the elasticity of substitution will be smaller in the course of the increase of income or population.[9] The problem remains whether the economic development (the increase of Y) and the larger size of economy (the increase of N) have the same qualitative influences upon the technical structure. Here we adopt the linear approximation of the coefficients with Y and N (17) and we may clarify the difference by more accurate treatment in the future.

IV. SUMMARY AND CONCLUSION

In this paper we specified and estimated a VES and VRS production function through the deductive method by introducing the size of economy variable to the Diwan-Dhrymes type reduced from equation. This specification was supported by the cross-section data of Latin American countries when we use population or income as the size variable. This way of generalization of the CES production function must be supplemented by various cross-checks, but the important point is that the elasticity of substitution and the economy of scale are subject to change simultaneously when we treat the aggregate production function of manufacturing industry.

REFERENCES

1. Bodkin, R. G., and Klein, L. R. "Non-linear Estimation of Aggregate Production Functions," *Review of Economics and Statistics*, Vol. 49, No. 1 (February 1967).
2. Dhrymes, P. J. "Some Extensions and Tests for the CES Class of Production Functions," *Review of Economics and Statistics*, November 1965.
3. Diwan, R. K. "Returns to Scale in Indian Industry: a Comment," *The Econometric Annual of the Indian Economic Journal*, Vol. 15, No. 4 (April 1968).
4. Fukuchi, T., and Hosono, A. *Estudio Sobre La Funccion de Produccion de La Industria Manufacturera en America Latina* (Santiago: CEPAL, 1970).
5. Maddala, C. S., and Kadane, J. B. "Estimation of Returns to Scale and the Elasticity of Substitution," *Econometrica*, July–October 1967.
6. Murata, Y. "Bias in the Estimation of the CES Class of Production Functions," *Economic Studies Quarterly*, Vol. 18, No. 2 (September 1967).
7. Nerlove, M. "Recent Empirical Studies of the CES and the Related Production Functions," in *The Theory and Empirical Analysis of Production* (New York: NBER, 1967).
8. Ranis, G., and Fei, J. "Innovation, Capital Accumulation, and Economic Development," *American Economic Review*, June 1963.
9. Sato, R., and Hoffman, R. F. "Production Functions with Variable Elasticity of Factor Substitution: Some Analysis and Testing," *Review of Economics and Statistics*, Vol. 50, No. 4 (November 1968).
10. Soskice, D. "A Modification of the CES Production Function to Allow for Changing Returns to Scale," *Review of Economics and Statistics*, November 1968.
11. Zellner, A., and Revanker, N. S. "Generalized Production Functions," *Review of Economic Studies*, April 1969.

[9] Technological change is caused either from changes in endowment of resources or from changes of demand structure for manufactured goods. Ranis-Fei [8] argued that the labor-saving type innovation occurs in the course of the diminishing unlimited supply of labor, and emphasized the supply side. In this paper we emphasize the change of the demand side. This is an interesting topic for future study.

APPENDIX TABLE I
CLASSIFICATION BY THE SIZE (NUMBER OF EMPLOYEES)

Group \ Country	Number of Employees	Brazil	Chile and Peru	Colombia	Costa Rica	Mexico
—	0 1–4	– 1–4	– –	– 1–4	– 1–4	– 1–5
I	5–9	5–9	5–9	5–9	5–9	6–25
II	10–19	10–19	10–14 15–19	10–14 15–19	10–19	
III	20–49	20–49	20–49	20–24 25–49	20–29 30–39 40–49	26–50
IV	50–99	50–99	50–99	50–74 75–99	50–59 60–69	50–
V	100–	100–249 250–499 500–999 1,000–	100–199 200–499 500–	100–199 200–	70–	

APPENDIX TABLE II
DATA OF SIZE VARIABLE

Country	Census Year	Income (10 Million Dollars)	Population (10 Million)
Brazil	1959	18.903	6.824
Colombia	1959	5.050	1.504
Chile	1951	4.663	0.723
Peru	1963	4.163	1.096

Note: All the units of the monetary data were converted to U.S. dollars using the Braithwaite estimates of exchange ratios: see, "The Measurement of Latin American Real Income in U.S. Dollars," *Economic Bulletin for Latin America*, October 1967.

APPENDIX TABLE III
REGIONAL DATA OF BRAZIL

Region \ Variable	Persons Per Establishment	Population (10^6)	Per Capita Income ($)	Total Income (10^6 $)
North & Central	6.87	5.30	120	634
North-east	10.02	15.17	92	1,387
East	13.22	23.91	205	4,889
South	15.17	23.66	334	7,898