The Frame of the Second Five Year Plan in India was constructed on a theoretical foundation given by Prof. P. C. Mahalanobis. However, apart from this practical importance of his theory, Prof. Mahalanobis' growth model, I think, has contributed to make a progress in the theory of economic growth. The contribution is considered as a generalization of Harrod-Domar's theory by introducing a saving function with an autonomous part of savings in addition to a part depending on national income level. That is, in Harrod-Domar model, the saving function has been considered to be

\[ S = aY, \]

where \( a \) stands for an average rate of savings. This relation means that a proportional part of national income is always saved. Instead, Mahalanobis model has introduced the following type of saving function

\[ S = sY + h, \]

where \( s \) stands for a marginal rate of savings and \( h \) for an autonomous part of savings. This relation implies that savings are composed of a part depending on national income level and an autonomous part.

For the saving function of Harrod-Domar type, we have a steady growth path of national income, on which national income grows at a certain constant rate. By introducing a saving function that was used in Mahalanobis model, we can expect an accelerated growth path of national income, where the accelerated growth implies an increasing

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growth rate. This accelerated growth path can include Harrod-Domar's steady growth path as a special case in which the marginal rate of savings is equal to the corresponding average rate.

In this paper, we shall concentrate our attention on the characteristics of the Mahalanobis' accelerated growth path. First of all, we explain the original Mahalanobis model (section 1). Next, we modify it in the light of criticisms of it given by some economists (section 2). By this process, we can see that the Mahalanobis' accelerated growth path is a generalized growth path of Harrod-Domar's line (section 3). And in the next stage, we examine some characteristics of the accelerated growth path (section 4). Lastly, we check a feasibility of the accelerated growth path in our actual situation by examining some qualifications of the saving function concerned with the Mahalanobis model (section 5).

I

The Mahalanobis model has been constructed in terms of Keynesian aggregates; national income, investment, savings, and consumption. Two sectors are considered in the model; production goods producing sector (K-sector) and consumption goods producing sector (C-sector). This sectoral classification is not for an intersectoral analysis of economy but for analysing an allocation of investment to respective sectors. Price situation is kept constant in his argument. And foreign trade is not considered in his model.

The fundamental assumptions in the Mahalanobis model are as follows:
(a) the saving-investment equilibrium is maintained, and
(b) the production processes in respective sectors are always operated under full capacity situation.

Investment at time \( t \) is divided into two parts; investments to K-sector and to C-sector. \( r_k \) and \( r_c \) stand for respective allocation ratios of investment to K-sector and to C-sector. We never do consider the

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1 These sectors are vertically aggregated within themselves. Sectors producing raw materials for consumption goods are aggregated with C-sector, and this rule is applied to K-sector as well. This is somewhat at variance with the Marxian schema of expanding reproduction where raw materials producing sectors are combined together with K-sector in the so-called Department I. G. A. Fel'dman, who studies a plan model for the Soviet Economic Planning from the viewpoint of Marxian economics, has constructed the very similar model to Mahalanobis model out of the Marxian schema of expanding reproduction. Cf. E. D. Domar, *Essays in the Theory of Economic Growth*, New York, Oxford University Press, 1957, Chap. 9.
other producing sectors, so that the sum of \( r_k \) and \( r_c \) naturally becomes unity. And let \( b_k \) and \( b_c \) be respective investment productivities in K-sector and in C-sector, where investment productivity means increment of income per one unit of investment in each sector. Sum of incomes in respective sectors becomes national income.

We have the following equation system, basing on these relations, which composes the original Mahalanobis model, that is

\[ Y_t = S_t + C_t, \]
\[ I_t = S_t, \]
\[ 1 = r_k + r_c, \]
\[ \Delta I_t = b_k r_k I_t, \]
\[ \Delta C_t = b_c r_c I_t. \]

The first relation shows balance equation of national income, and the second relation expresses the saving-investment equilibrium condition.\(^1\) The third relation shows an allocation of investment to respective sectors. And the last two relations stand for relations between investment and increments of income in respective sectors under full capacity situation.

In this equation system, \( b_k \) and \( b_c \) are parameters which are exogenously given from the outside of this system. Initial values of national income and investment, \( Y_0 \) and \( I_0 \), are assumed to be given by the initial conditions. When allocation ratio of investment to K-sector, \( r_k \), is given as a policy instrument to determine the degree of industrialization, national income, investment, savings, and consumption at time \( t \), then \( Y_t, I_t, S_t \) and \( C_t \) and allocation ratio of investment to C-sector, \( r_c \), are uniquely determined by the simultaneous equations.

By solving the simultaneous equations, we have the following relations as to national income and investment at time \( t \),

\[ I_t = I_0 (1 + b_k r_k)^t, \]
\[ Y_t = Y_0 \left[ 1 + a_0 \left( \frac{b_k r_k + b_c r_c}{b_k r_k} \right) \left( (1 + b_k r_k)^t - 1 \right) \right]. \]

Relation (2) shows a growth path of investment over time, where the rate of growth is \( b_k r_k \). This rate of growth is constant over time for the given allocation ratio of investment to K-sector, \( r_k \). And relation (3) is a fundamental equation of national income growth in the Mahalanobis model, the first two equations were combined together and described in terms of increments of income in respective sectors, that is

\[ \Delta Y_t = \Delta I_t + \Delta C_t. \]

This shows a composition of national income increment. Such a change in expression does never mean a change of Mahalanobis' original ideas.

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\(^1\) In the original Mahalanobis model, the first two equations were combined together and described in terms of increments of income in respective sectors, that is

\[ \Delta Y_t = \Delta I_t + \Delta C_t. \]
Mahanalobis’ Growth Model

In the fundamental equation of national income growth, we can see the rate of growth of national income at time $t$ to be

$$\frac{dY_t}{Y_t} = \alpha t,$$

where $\alpha$ stands for an average rate of savings at time $t$ and $b$ for a global investment productivity of the economy as a whole. This global investment productivity is defined as a weighted average of sectoral investment productivities,

$$b = b_k r_k + b_c r_c,$$

where the weights are allocation ratios of investment to respective sectors. In this relation (5), the global investment productivity is determined for the given allocation ratio of investment to K-sector, because the sectoral investment productivities are parameters given from the outside of this system and the allocation ratio of investment to C-sector is given for the given allocation ratio of investment to K-sector. So the rate of growth of national income at time $t$ depends only on the average rate of savings at time $t$, which is expressed as follows:

$$a_t = a_{t-1} \cdot \frac{1 + \frac{b_k r_k}{b_k r_k + b_c r_c} \cdot b}{1 + a_{t-1} b}.$$

By solving this difference equation, we have

$$a_t = a_0 \cdot \frac{(1 + b_k r_k)^t}{1 + a_0 \left( \frac{b_k r_k + b_c r_c}{b_k r_k} \right) (1 + b_k r_k)^t - 1}.$$

This shows that the time path of average rate of savings converges to certain value, which is given for the given allocation ratio of investment to K-sector, for any initial value of average rate of savings.

$$\lim_{t \to \infty} a_t = \frac{b_k r_k}{b_k r_k + b_c r_c}.$$

The relation (6) implies that (a) when $\frac{b_k r_k}{b_k r_k + b_c r_c}$ is larger than $a_{t-1}$, $a_t$ is always larger than $a_{t-1}$, and (b) when $\frac{b_k r_k}{b_k r_k + b_c r_c}$ is smaller than $a_{t-1}$, $a_t$ is always smaller than $a_{t-1}$. So it becomes clear from the relation (7) and (8) that (a) when $\frac{b_k r_k}{b_k r_k + b_c r_c}$ is larger than $a_0$, $a_t$ is steadily increasing and converging to $\frac{b_k r_k}{b_k r_k + b_c r_c}$, and (b) when $\frac{b_k r_k}{b_k r_k + b_c r_c}$ is smaller than $a_0$, $a_t$ is steadily decreasing and converging to the same
The most important characteristics of the growth path of national income is as follows: Under the assumption of constant investment productivities in respective sectors, in order to keep an increasing rate of growth of national income, average rate of savings at time $t$ must be larger than that at time $t-1$. And for maintaining the situation, value of $\frac{b_kr_k}{b_kr_k + b_0r_c}$ has to be larger than the average rate of savings at time $t-1$. For this purpose, Prof. Mahalanobis proposed to keep $r_k$ as high as possible within a permissible limit of economy. This was one of his ideas for making the plan frame of the Second Five Year Plan in India.
Mahalanobis' Growth Model

For examining an effectiveness of \( r_k \) as policy instrument, we have

\[
\frac{d}{dr_k} \left( \frac{b_k r_k}{b_k r_k + b_c r_c} \right) = \frac{b_k b_c}{b_k^2} > 0.
\]

This means that if the value of \( r_k \) is increased by any means, and if the value of \( \frac{b_k r_k}{b_k r_k + b_c r_c} \) is kept at any level to be higher than the value of \( a_{t-1} \), an increasing rate of growth of national income can be maintained on the growth path shown by (3).

In the fundamental equation of national income growth, \( a_0 \) is given by the initial conditions, and \( b \) depends on the allocation ratio of investment to K-sector for given investment productivities in respective sectors.\(^1\) Considering the relation (5), the higher \( r_k \), the higher \( b \) for \( b_k > b_c \) and the lower \( b \) for \( b_k < b_c \). So in case \( b_k > b_c \), the initial rate of growth of national income \((a_0 b)\) is increased by the higher \( r_k \). On the other hand, in case \( b_k < b_c \), the initial rate of growth is decreased by the higher \( r_k \).\(^2\) We consider the case \( b_k < b_c \) is feasible in the actual situation. An increment of income per unit of investment may be smaller in K-sector than in C-sector.\(^8\) In this feasible case, the higher \( r_k \) brings about the lower rate of growth of national income at initial time. However, the higher \( r_k \) accelerates the rate of growth of national income over time. This acceleration of the rate of growth can sufficiently cover the loss at the initial stage.

II

Many comments and criticisms have been concentrated on the Mahalanobis’ theory of economic planning. The most important criticism was concerned with an absence of the demand-side consideration of products in his theory.\(^4\)

As we have shown above, the Mahalanobis model of economic growth has been composed of the relations included in (1). The first relation is balance equation of national income. The second equation

1. \( b \) is also depending on \( r_c \). But, as we have the relation \( 1 = r_k + r_c \), \( r_c \) is also depending on \( r_k \).
2. The case \( b_k = b_c \) excludes these considerations.
3. Prof. Mahalanobis takes the following estimates:
   \[ b_k = 0.20 \text{ and } b_c = 0.35 \sim 1.25. \]
4. This criticism was given by Prof. S. Tsuru (“Some Theoretical Doubts on the Plan Frame,” Economic Weekly, Annual Number, 1957) and Prof. A. K. Sen (“A Note on the Mahalanobis Model of Sectoral Planning,” Arthaniti, 1958, Vol. 1, No. 2).
shows the saving-investment equilibrium. The third relation is definitional relation of allocation of investment to respective sectors. And the last two equations express the supply-side conditions of products in respective sectors, which are the relations between investment and increments of income in respective sectors under full capacity situation. Thus, the Mahalanobis model does neglect to include the demand-side conditions.

In order to introduce the demand-side conditions of products to the Mahalanobis model, let us consider the following relation:
\[
S_t = sY_t + h.
\]

This means that savings at time \( t \) is composed of a part depending on national income level at time \( t \) and an autonomous part. \( s \) stands for marginal rate of savings.

The second equation in (1) stands for the saving-investment equilibrium, so the relation (9) shows the demand-side condition of product in K-sector.

So long as the system includes the second equation in (1), the relation (9) also means the following relation:
\[
C_t = (1 - s)Y_t - h.
\]

This implies that consumption at time \( t \) is composed of a part depending on national income level at time \( t \) and an autonomous part. \((1 - s)\) stands for marginal rate of consumption. The relation (10) expresses the demand-side condition of product in C-sector. However, in a system including the second relation in (1) and the relation (9), the relation (10) is not independent from them. So it is not necessary to introduce the relation (10), in addition to considering the relation (9), in examining the demand-side conditions of products in the Mahalanobis model.

Thus the modified Mahalanobis model of economic growth is shown by the following equation system:
\[
\begin{align*}
Y_t &= S_t + C_t, \\
S_t &= sY_t + h, \\
I_t &= S_t, \\
1 &= r_kb_k + r_c b_c, \\
\Delta I_t &= b_k r_k I_t, \\
\Delta C_t &= b_c r_c I_t.
\end{align*}
\]

In this system, the saving-investment equilibrium and the full capacity situation are assumed. The second relation shows the saving function. Other relations are the same as those including in the equation system (1).

In these simultaneous equations, \( b_k, b_c \) and \( h \) are parameters given
by the exogenous conditions. \( r_e \) is given as policy instrument due to planner's decision. Thus we have six unknowns \((Y_t, I_t, S_t, C_t, s, r)\) for six equations included in (11). When the initial levels of national income and investment are given, these six unknowns are uniquely determined by the six equations for the given \(Y_0\) and \(I_0\).

By solving the simultaneous equations (11), we have
\[
\begin{align*}
(12) \quad I_t &= I_0(1 + sb)^t, \\
(13) \quad Y_t &= Y_0 \left[ 1 + \frac{a_0}{s} \{(1 + sb)^t - 1\} \right], \\
(14) \quad s &= \frac{b_k r_k}{b_k r_k + b_c r_c},
\end{align*}
\]
where
\[
b = b_k r_k + b_c r_c.
\]

Relation (14) shows the marginal rate of savings under the saving-investment equilibrium situation of the system; we call this the required marginal rate of savings.\(^1\) In the Mahalanobis' growth model without the demand-side condition, this relation was neglected.

Relation (12) shows a growth path of investment over time, where the growth rate of investment is \(sb\). As we have already shown, the growth path of investment over time is expressed by the relation (2), where the growth rate of investment was \(b_k r_k\). However, let us consider the following relation,
\[
(15) \quad b_k r_k = \frac{b_k r_k}{b_k r_k + b_c r_c} \cdot b = sb.
\]
This means that there is no difference between the relations (2) and (12).

\(^1\) S. Chakravarty (The Logic of Investment Planning, Amsterdam, North Holland Publishing Co., 1959) has also proved this from the other point of view. His proof is roughly explained as follows: Considering the relation of investment productivity in K-sector, we have
\[
b_k r_k = \frac{\Delta I_t}{I_t}.
\]
And from the definitional relation of global investment productivity, we have
\[
b_k r_k + b_c r_c = \frac{\Delta Y_t}{Y_t}.
\]
So, consequently, we have
\[
\frac{b_k r_k}{b_k r_k + b_c r_c} = \frac{\Delta I_t}{I_t}, \quad I_t = \frac{\Delta I_t}{\Delta Y_t} \cdot \frac{\Delta Y_t}{Y_t}.
\]
Introducing the saving-investment equilibrium condition, we have
\[
\Delta I_t = \Delta S_t,
\]
so the above relation becomes
\[
\frac{b_k r_k}{b_k r_k + b_c r_c} = \frac{S_t}{Y_t} = s.
\]
Relation (13) shows a time path of national income growth. And we have already the time path of national income growth in the Mahalanobis' growth model expressed by the relation (3). Considering the relations (14) and (15), it is certified that there is no difference between the relations (3) and (13).

III

Contribution of Professors R. F. Harrod and E. D. Domar to the theory of economic growth has been to show that a steady growth of national income depends on an average rate of savings and an investment productivity.¹

The Harrod-Domar model of economic growth has also been constructed in terms of Keynesian aggregates. But this model does not include any sectoral classification. The assumptions of constant price situation and no foreign trade are also maintained in this model.

The most important assumptions in the Harrod-Domar model are as follows:
(a) the saving-investment equilibrium is maintained,
(b) the production process is always operated under full capacity situation, and
(c) the marginal rate of savings is always equal to the corresponding average rate of savings.

The first two assumptions are the same as in the Mahalanobis model. The third assumption is the very point by which the Harrod-Domar model is distinguished from the Mahalanobis model.

The above-mentioned assumption (c) implies that the Harrod-Domar


The expression of the Harrod-Domar model (16) is mainly dependent upon Harrod's original idea. Domar has presented his growth model in the following form,

\[ bI_t = dY_t, \]
\[ dS_t = sY_t, \]
\[ dI_t = dS_t. \]

This system differs somewhat in expression from his original equation system, but the idea is not damaged.

And this system has the same characteristics as the system (18) has. So, solving the system, we can expect the same growth paths of national income and investment as shown by (19) and (20). However, as Domar has assumed that the marginal rate of savings is always equal to the average rate of savings, he did not follow the growth paths shown by (19) and (20). So his growth model has been treated together with the Harrod's growth model.
Mahalanobis' Growth Model

model takes the saving function by which a proportional part of national income is always saved. Thus the model is constructed as follows:

\[ \Delta Y_t = b I_t, \]
\[ S_t = a Y_t, \]
\[ I_t = S_t, \]

where \( Y_t, I_t \) and \( S_t \) stand for national income, investment, and savings at time \( t \), \( a \) for average rate of savings, and \( b \) for global investment productivity as a whole of economy.

The first relation in (16) shows that a certain amount of investment produces \( b \) times of increment of national income under the full capacity situation. The second relation expresses that a certain portion of national income is always saved. And the third relation stands for the saving-investment equilibrium.

In the equation system (16), we have three unknowns \( (Y_t, I_t \) and \( S_t) \) for three equations. And these unknowns are uniquely determined for the parameters \( (a \) and \( b) \) and the initial conditions \( (Y_0 \) and \( I_0) \). Then we have:

\[ Y_t = Y_0(1 + ab)^t, \]
\[ I_t = I_0(1 + ab)^t, \]
\[ S_t = I_0(1 + ab)^t. \]

In this case, all the variables are growing at the same rate, which is the steady growth.

So long as an average rate of savings is kept in constant, the corresponding marginal rate of savings must be constant. And this marginal rate is equal to the average rate. That is, when the relation \( S_t = a Y_t \) is maintained, we have

\[ \Delta S_t = a \Delta Y_t, \]

where \( a \) stands for the marginal rate of savings and the value of this rate is equal to the average rate. Thus, the steady growth is maintained only in the case that the marginal rate of savings is always kept in constant and is equal to the average rate.

Now let us remove the assumption \( (c) \) in the Harrod-Domar model. This means that we consider the following saving function

\[ S_t = s Y_t + h, \]

which is the same saving function as we used for modifying the Mahalanobis' growth model. In this case, the Harrod-Domar model (16) is modified as follows:

\[ \Delta Y_t = b I_t, \]
(18) \[ S_t = sY_t + h, \]
\[ I_t = S_t. \]
In this system, we have three unknowns \((Y_t, I_t, \text{and} S_t)\) for three equations. These three unknowns are uniquely determined for the parameters \((b, s, \text{and} h)\) and the initial conditions \((Y_0, \text{and} I_0)\). So we have

(19) \[ Y_t = Y_0 \left[ 1 + \frac{a_0}{s} \{(1 + sb)^t - 1}\right], \]
(20) \[ I_t = I_0 (1 + sb)^t. \]
Relation (19) shows a growth path of national income over time, and relation (20) expresses a growth path of investment over time. These growth paths are formally the same as those shown in the modified Mahalanobis model. The differences between the modified Harrod-Domar model and the modified Mahalanobis model are explained as follows: In the modified Harrod-Domar model, the marginal rate of savings is given from the outside of the system. Contrarily, in the modified Mahalanobis model, it is determined for the given value of allocation ratio of investment to K-sector. This is only due to the fact that Mahalanobis wanted to analyse the effects of investment allocation to economic growth but Harrod-Domar did not. In this paper, we would like to disregard the problem of investment allocation and only to analyse some characteristics of those growth paths. In this respect, we find the modified Harrod-Domar path to be the same as the Mahalanobis path. More explicitly, the Mahalanobis path is a generalized growth path on Harrod-Domar line.

IV

Even in the case of constant marginal rate of savings, which has different value from the corresponding average rate of savings, investment can grow at a constant rate. In relation (20), which shows a growth path of investment over time, we have the rate of growth to be

(21) \[ \frac{\Delta I_t}{I_t} = sb. \]
However, as we can see in the relation (19), the rate of growth of national income is not constant over time. We have

(22) \[ \frac{\Delta Y_t}{Y_t} = a_t b, \]
(23) \[ a_t = a_{t-1} \cdot \frac{1 + sb}{1 + a_{t-1} b}. \]
The rate of growth of national income at time \(t\) is determined by an
average rate of savings at time \( t \) and an investment productivity. From the relation (23), we can see the following characteristics of the average rate of savings:

\[
(24) \quad a_t = \begin{cases} 
\text{increasing} & \text{for } a_{t-1} < s, \\
\text{constant} & \text{for } a_{t-1} = s, \\
\text{decreasing} & \text{for } a_{t-1} > s.
\end{cases}
\]

This means that, in order to maintain an increasing rate of growth of national income, the marginal rate of savings must be larger than the average rate of savings at the corresponding time period.

When we consider the relation

\[
s = \frac{b_k r_k}{b_k r_k + b_or_c},
\]

all the arguments developed in connection with the Mahalanobis' growth path of national income are true here again.

Solving the difference equation (23), we have a time path of average rate of savings:

\[
(25) \quad a_t = a_0 \cdot \frac{(1 + sb)^t}{1 + \left( \frac{a_0}{s} \right) \{(1 + sb)^t - 1\}}.
\]

This time path always leads the average rate of savings to a certain finite level, that is the marginal rate of savings,

\[
(26) \quad \lim_{t \to \infty} a_t = s.
\]

We can expect the same result whether the average rate of savings at the initial stage is larger than the marginal rate of savings or smaller.

Considering such a movement of the average rate of savings over time, we can apply the same type of movement to the growth rate of national income. When the initial value of the average rate of savings is smaller than that of the marginal rate of savings, the growth rate is acceleratedly increasing and converging to certain constant level, which is the product of the marginal rate of savings and the investment productivity. When both are the same, the growth rate is constant and the national income steadily grows. When the initial value of the average rate of savings is larger than the marginal rate of savings, the growth rate is acceleratedly decreasing and converging to the value of product of the marginal rate of savings and the investment productivity.

\[
\n\]

We are now in a position to examine the feasibility of these three types of growth path of national income over time. For this purpose,
we have to examine some qualifications of the saving function that we have now taken in our arguments, that is
\[ S_t = sY_t + h. \]
In this saving function, we have assumed the marginal rate of savings and the autonomous part of savings to be kept in constant.

The average rate of savings is expressed as follows:
\[ a_t = \frac{S_t}{Y_t} = s + \frac{h}{Y_t}. \]
This means that (a) \( h \) being positive, the average rate of savings is larger than the marginal rate of savings and decreases with increasing national income level, (b) \( h \) being zero, both rates are always equal to each other and kept constant, and (c) \( h \) being negative, the average rate of savings is smaller than the marginal rate of savings and increases with increasing national income level.

![Figure 2](image_url)

These situations are graphically explained by Figure 2. In Figure 2, the horizontal axis is in terms of national income and the vertical axis in terms of savings. Let us picture three types of saving functions on
Figure 2; $S_1$ with positive $h$, $S_2$ with negative $h$ and $S_3$ with null $h$. On the saving function $S_1$, the average rate of savings is larger than the marginal rate of savings and increases with increasing national income level. Contrarily, on the saving function $S_2$, the average rate of savings is smaller than the marginal rate of savings but increases with increasing national income level. And on the saving function $S_3$, the average rate of savings is always equal to the marginal rate of savings and be kept in constant over time. Like this, the positive, null and negative values of $h$ can correspond to the accelerated, steady and decelerated growths of national income respectively.

Economic meaning of $h$ is to express a negative value of the fundamental consumption, which means consumption level at zero level of national income. From this point of view, in a normal situation of economy, $h$ takes, at most, non-positive value. We cannot imagine a negative fundamental consumption. In this respect, we can restrict our argument of national income growth within the accelerated and, at least, the steady growth of national income over time. This means that we give up the case with positive $h$.

Thus we can see the feasibility of the accelerated and the steady growth of national income over time, and the infeasibility of the decelerated growth.

VI

The arguments on Mahalanobis line make it clear that the accelerated growth of national income over time is the general case of national income growth. This includes the steady growth as a special case with a null fundamental consumption. We think this is one of the most important merits of the Mahalanobis’ theory of economic growth.