The Developing Economies, XXXIX-2 (June 2001): 143–67

A WORLD EQUATION

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I. INTRODUCTION

THE current world economy is made up of more than 180 countries, and is rapidly changing as the world population grows and a number of developing countries approach developed status. Given the current trend, how will the world economy look in the long run? Will there be a big thrust from Asia as China and India increase their economic shares? This paper tries to sketch a long-run view of the world economy and forecast the portions that different countries will account for.

Each national economy grows over time based on the natural increase of production factors such as labor and capital. These economies engage in international trade, and exchange their products to increase their welfare. Also the production factors are moving between countries seeking higher rates of remuneration. As a result, a country's GDP as a share of world GDP changes over time. When the initial shares of world GDP are given, could there be a model that transforms the initial shares into long-run GDP shares? Were such a model to be constructed and reduced to an equation, we could legitimately regard it as a "world equation," which would summarize the whole history of the world economy.

This paper constructs such a world model and resulting world equation based on the scheme of the PVU-economy model. This model treats a multi-country economy with a (1 good, m factors, n countries) background (where m or n is any positive integer), and a special version of the broader model family of the R-economy model (Fukuchi 2000a). This model has three features: (i) it explicitly describes the linking of n-economies, (ii) it shows the shares of world GDP numerically, (iii) it calculates the long-run values. Therefore, this model can be regarded as an n-countries-related digital-type long-run growth model (NDLG-model). To treat the world economy in a wide scope and long-run manner, the model adopts many bold assumptions. But while paying a big cost in simplification, it can show a long-run image of the world economy in a similar way that the Hubble telescope shows the further distances of the universe. After constructing the model, I will apply it empirically to the data of 1990 and 1996 for thirty-two countries to clarify transitory dynamics and the long-run equilibrium of the world economy. I will estimate basic movement elasticity with a final test, and make simulation of the future. The model shows potential for a strong China-India thrust with the two countries increasing their shares of world GDP in the long run. I will also discuss the effects of a successful control of the population explosion on the transitory dynamics of the world economy.

PVU-economy modeling proposes new approaches in three areas: (i) The world economy is a multiregional and multiindustry economy, and can be analyzed either from the demand or supply side, either in terms of multiregion or multiindustry, and either in the short or long run. The traditional Interindustry Analysis (IIA, based on Leontief input-output model) relies on the demand-side short-run approach. The PVU-economy model proposes Interregional Analysis (IRA, based on the Fukuchi R-Economy model [Fukuchi 2000a]), which offers a supply-side long-run approach. (ii) The traditional international economics often adopted the Ricardian (2, 2, 2) scheme of two goods, two factors, and two countries. The PVU-economy model proposes the use of a (1, m, n) scheme. While it pays a high price in simplification by assuming only one good, it can generalize the number of factors and countries to any positive integer, and can also discuss the long-run equilibrium situation. (iii) The argument over convergence has depended heavily on the Solow-type neoclassical growth model, and has tried to define convergence speed based on labor productivity. But this approach has two intrinsic weakness: (a) it neglects the relations among countries, and (b) the impossibility of defining convergence speed when long-run labor productivity diverges. When productivity reaches to a constant, the speed can be defined rigorously, while each country must reach at a specific level, so the actual implication is weak. The PVU-economy model covers these two weaknesses by explicitly considering factor movement among countries, and rigorously defining the convergence speed based on long-run shares of world GDP.

Fukuchi (1996) first developed the basic methodology of the NDLG-model, and applied it to data for nine regions of Japanese economy. Fukuchi (2000a) defined a family of multiregional models (R-economy model), and rigorously discussed the PCU-economy (constant-return-to-scale Cobb-Douglas function and normalized distance). Fukuchi (2000b) extended the discussion in three directions. This paper contains the discussion of the PVU-economy (Cobb-Douglas function with economies of scale and normalized distance), and its application to the current world economy.

Following this Introduction, Section II summarizes the basic trend of the world economy, Section III presents the basic model, Section IV discusses the uniqueness of mapping by world equation, Section V presents the empirical results of this study based on the data of 1990 and 1996 for thirty-two countries, and Section VI presents the conclusions.

II. MODERN WORLD ECONOMIC SYSTEM

According to McEvedy and Jones (1978), (a) the world population was 14 million in 3000 B.C., 50 million in 1000 B.C., 265 million in A.D. 1000, and 5,000 million in A.D. 2000; (b) the state of the world system at each of those times was: nonexistent, premodern, modern, and postmodern; (c) the state of socioeconomic interaction was: nonexistent, low, medium, and high. After A.D. 1500, the discovery of the American continent, and the opening of direct sea routes between Europe, India, and China resulted in a quick expansion of the mutual interaction between continents, and in the construction of a global system (Modelski 1972, Chap. 3). In the twentieth century, the development of the radio, airplanes, and satellite communication systems have integrated the world economy further. After World War II, colonial empires broke up into independent states. So the world system and its economy has a 500-year history at most, and only 50 years in its current form.

The 500 years of world system history has had a number of long-run hegemonistic cycles during which a dominant state prevailed. The world seas have been dominated by Portugal (1502–1544), the Netherlands (1608–1642), England (1719–1723, 1809–1890), and now the United States (1944–) (Modelski 1987, Table 2.2); and big wars marked the changes in hegemony. The size of each hegemonistic state became bigger in relative and absolute terms. The size of population and the share of world GDP when each country came to the fore was: Portugal (1.25 million, 0.30 per cent, 1500), the Netherlands (1.50 million, 0.30 per cent, 1600), England (9.25 million, 1.50 per cent, 1700 and 16.00 million, 1.80 per cent, 1800) and the United States (100.00 million, 5.40 per cent, 1914) (Modelski 1987, Table 2.5).

When we reckon the rise and fall of hegemonistic countries in the world, we notice two facts: (1) the short history of each country's hegemony, and (2) the overwhelming effects of immigration.

After Columbus reached America in 1492, a massive of people emigrated voluntarily or involuntarily from Europe and Africa to the sparsely populated new continents. In 2000 Brazil celebrated the 500th anniversary of the first immigrants from Portugal. Thus the countries of the Americas have lived less than 500 years, and many colonies in other parts of the world have gained independence only since World War II.

According to an OECD report (OECD 1995), the ratio of foreigners to total population increased in many member countries between 1983 and 1993: Luxemburg 26.3% to 31.1%, Switzerland 14.4% to 18.1%, Austria 3.9% to 8.6%, Germany 7.4% to 8.5%, Sweden 4.8% to 5.8%, the Netherlands 3.8% to 5.1%, Norway 2.3% to 3.8%, and Denmark 2.0% to 3.6%. If the ratio in these countries were to increase by the rate of 1% in 10 years, and if this trend were to continue into the future, the ratio would reach 100% in 20 years in Luxemburg and Austria, in 30 years in Switzerland, in 100 years in the Netherlands, Germany, Sweden, and Norway.

Thus if the current trend of international resource movement continues, there will be big changes in country shares of world GDP even if the current political scheme is maintained.

In the twentieth century, the most remarkable change of a country's share of world GDP was that for Japan. Between 1870 and 1970, per capita income increased by eighteen times in Japan compared with three times for Australia, eight times for the United States, and nine times for Germany. Thus catch-up and convergence has occurred among the currently industrialized countries. This trend can be understood within the framework of the catch-up thesis which rests on the hypothesis that each country shares common technology, and a low-wage country can realize a cheaper cost of production, higher export growth, and resulting higher economic growth. Let us assume that: (1) technology is a common international public good, (2) production technology is of the Cobb-Douglas type, and (3) the mobility of capital is higher than that of labor. Then a country with lower capital intensity of labor is a low-income country but has a higher capital productivity, and attracts more capital inflow than labor outflow. In this case, a low-income country will realize higher economic growth. With such an understanding in mind, let us construct a basic framework in the next section.

III. CONSTRUCTION OF A BASIC PVU-ECONOMY MODEL

Let us consider an *n*-countries world economy. A two-country model is simplistic and less interesting because it suffices to determine the movement of one country, then the other country can be determined automatically as the residual. I will specify that $n \geq 3$, then the dynamics of *n*-economy ($n \geq 3$) is far more complex than that of a two-country model, and is similar to that of physical science.[1]

Fukuchi (2000a) constructed a family of multiregional models (called an R-economy) which has eight models according to three criteria: the production function is Cobb-Douglas (P) or CES (Q), economy of scale exists (V) or linear homogeneous (C), interregional distances are normalized (U) or not (D). In this paper I adopt the scheme of a PVU-economy with two factors.

I will suppose a multiregional economy with *n*-regions ($n > 2$) and two production factors (labor and capital). Each factor is imperfectly mobile between regions.[2]

[1] When there are three bodies, it is difficult to analytically determine the dynamic movement of the system when the initial position and speed of each body is specified. Recently simulation experiments using supercomputers clarified that the system shows a chaos-type movement.

[2] Therefore the treatment differs from that of international trade theory (ITT). In ITT, factor accumulation and international factor movement are discussed separately, and the analysis of factor movement usually rests upon the factor price equalization (FPE) condition if without the interference by tariffs; for example, Kemp (1969, p. 208), Takayama (1972, p. 395). So the analysis is partial and short-term. In the PVU-economy model, factor movement and accumulation go hand in hand, and

Each region produces output based on the common Cobb-Douglas production function with or without economy-of-scale and Hicks-neutral technological progress. Suppose that the ith region has endowment of labor (E_i) and capital (K_i), and produces output (Y_i). All the variables are defined as functions of time (T), but the time symbol is conveniently suppressed. $R(X)$ denotes the growth rate of X. Then PVU(2, n)-model can be specified as follows.

Multiregional PVU(2, *n*)*-economy*

- Exogenous natural growth rates of capital (R_k) and labor (R_e), where $R_e < R_k$.
- Cobb-Douglas production function:

$$\ln(Y_i) = \ln(B_o) + V \cdot (B_k \ln(K_i) + B_e \ln(E_i)) + PP \cdot (T),$$
$$B_e + B_k = 1,\ V \geq 1, \quad i = 1, \ldots, n. \qquad (3\text{-}1)$$

- Interregional factor mobility function:

$$E_{ji} = A_e(E_j E_i/E) \cdot \ln[(Y_i/Y_j)/(E_j/E_i)^{(1/V)}], \quad i, j = 1, \ldots, n,\ i \neq j, \qquad (3\text{-}2)$$
$$K_{ji} = A_k(K_j K_i/K) \cdot \ln[(Y_i/Y_j)/(K_j/K_i)^{(1/V)}], \quad i, j = 1, \ldots, n,\ i \neq j, \qquad (3\text{-}3)$$

- Bourguignon's inequality measure:

$$LE = -\sum(SE_i)\ln[(SY_i)/(SE_i)^{(1/V)}], \qquad (3\text{-}4)$$
$$LK = -\sum(SK_i)\ln[(SY_i)/(SK_i)^{(1/V)}], \qquad (3\text{-}5)$$
$$LY = B_e LE + B_k LK. \qquad (3\text{-}6)$$

- Other definitions:

$$K_i = (1 + R_k)K_i(t-1) + \sum K_{ji}, \quad i = 1, \ldots, n, \qquad (3\text{-}7)$$
$$E_i = (1 + R_e)E_i(t-1) + \sum E_{ji}, \quad i = 1, \ldots, n, \qquad (3\text{-}8)$$
$$\ln(Y_a) = \ln(B_o) + V \cdot (B_k \ln(K) + B_e \ln(E)) + PP \cdot T,$$
$$B_e + B_k = 1,\ V \geq 1. \qquad (3\text{-}9)$$
$$E = \sum E_j,\ K = \sum K_j,\ y_i = Y_i/Y_a,\ x_i = Y_i/K_i,\ y_a = Y_a/E,\ SE_i = E_i/E,$$
$$SK_i = K_i/K,\ SY_i = Y_i/Y_a,\ \sum(SY_i) = \text{or} < 1, \quad i = 1, \ldots, n. \qquad (3\text{-}10)$$

PP is the rate of neutral technological progress. V is the economy-of-scale parameter. T is year. E_{ji} and K_{ji} are the movement of labor and capital from jth to ith country. A_e and A_k are the mobility parameter of labor and capital, and can be understood approximately as movement elasticity (Fukuchi [2000a], p. 23). Y_a is an aggregate GDP when all resources are inputted together, so it differs from the mere sum of country GDP. In this framework, the technological intercountry differences, if they exist, are attributed to the differences in endowment of capital stock between

FPE is realized only in the long run. IBRD (1997) indicates the strong private capital movement in recent years.

countries. Inequality measures (3-4)–(3-6) are defined in an entropy type form, and play vital roles in the functioning of an economy. The entropy-type form was adopted in many inequality or similarity measures in the past.[3] In PVU-economy modeling, it was induced from a factor movement equation, and represents the growth of the auxiliary variable, PH, in (3-11) and (3-12).[4]

This PVU(2, n)-model contains $8n$ variables (Y_i, E_i, K_i, SE_i, SK_i, SY_i, y_i, x_i), $n(n-1)$ variables (E_{ji}, K_{ji}) and 6 variables (Y_a, E, K, LE, LK, LY), and consists of the same number of equations. For example, when the number of countries is 32 (like the empirical study in Section V), the total number of variables amounts to 1,254. Nine parameters (B_o, B_e, B_k, V, PP, A_e, A_k, R_e, R_k) determine the functioning of the model, but only five parameters (B_e, B_k, V, A_e, A_k) play roles in determining the long-run equilibrium shares of world GDP. When the initial conditions ($Y_i(0)$, $K_i(0)$, $E_i(0)$) are given, the model can tell us the future development path of this n-countries world. I summarize the result, which can be proved as follows:

THEOREM 1. *In the multi-country PVU(2, n)-economy,*

(i) Labor productivity diverges by the assumption: $R_e < R_k$.

(ii) Bourguignon's LE-index converges to zero, and assures factor price equalization (FPE) in the long run.

(iii) The long-run shares of factors and of output ($SE_i(\infty)$, $SK_i(\infty)$, $SY_i(\infty)$) *approach*

$$SE_i(\infty) = SK_i(\infty) = SY_i(\infty) = PH_i(0)^{(1/c)}/(\sum PH_j(0))^{(1/c)}, \quad i = 1, \dots, n, \qquad (3\text{-}11)$$

$$PH_i(t) = [(SE_i(0))^{(Be/Ae)} \cdot (SK_i(0))^{(Bk/Ak)}]/[(\sum SE_j(0))^{(Be/Ae)} \cdot (SK_j(0))^{(Bk/Ak)}], \quad i = 1, \dots, n. \qquad (3\text{-}12)$$

(iv) The speed of inequality convergence cannot be defined because world and country productivity diverge. But the average convergence speed of country output share ($DSY_i(T)$) *can be defined using the long-run shares.*

$$DSY_i(T) = (1/T)\ln[\ln(SY_i(0)/SY_i(\infty))/\ln(SY_i(T)/SY_i(\infty))]. \qquad (3\text{-}13)$$

Theorem 1 asserts that per capita income converges when the multi-country economy is described by the PVU-economy setting, independent of natural growth rates of production factors. The share convergence speed can also be defined for the country share of each production factor. Some comments are in order here. Fukuchi (2000a, p. 11, Theorem 1) provided the rigorous proof for the PCU-economy where $V = 1$. When $V > 1$, we redefine $(Y_i)^{(1/V)}$ as the new output variable. Then the production function is linear for new output, and all the contents of the theorem for the PCU-economy hold. This discussion suggests that the long-run share distribution will become more skewed when V becomes bigger.

[3] For example, Love (1986) adopted a similar measure as a concentration measure.

[4] See Fukuchi (2000a), p. 9.

The theory of international economics traditionally adopted the scheme of two commodities, two factors, and two countries, the so-called Ricardian (2, 2, 2) setting, which was very useful for analyzing international trade in the short run. The PVU-economy proposes another paradigm of one commodity, m factors and n countries, or (1, m, n) setting, which is useful for discussing the long-run incidence of factor movement. While this new scheme pays a high price in simplification by assuming only one commodity, it generalizes the number of factors and countries to any positive integer.

IV. WORLD EQUATION AND EIGENVECTOR

Let us write the endowment of kth factor in ith region as X_i^k ($k = 1, \ldots, m$; $i = 1, \ldots, n$). In this section the number of factors will not necessarily be two, so the suffix k is used to denote the factor's number.

There is a multi-country world with n members, and the output share of ith member (SY_i) is initially given ($SY_i(0)$). We define the share vector as (prime (′) implies transpose):

$$SY = (SY_1, SY_2, \ldots, SY_n)'. \tag{4-1}$$

If the long-run share $SY_i(\infty)$ is analytically calculable, we call the many-to-one mapping (F) from $SY(0)$ to $SY(\infty)$ as world mapping or as an equation which summarizes the whole history of the world economy,

$$F(SY(0)) = SY(\infty). \tag{4-2}$$

When an initial positive output share vector (SY^*) exists as a point in the n-dimension simplex with coordinates (4-1), and satisfies

$$F(SY^*) = SY^*, \tag{4-3}$$

then we can regard SY^* as the eigenvector of world output distribution. When the law of eqi-marginal-productivity or factor-price-equalization (FPE) holds from the beginning, we can call it a trivial FPE solution. Instead, we will look for a world equation and accompanying non-trivial eigenvector.

1. *A world economic system*

Each ith country ($i = 1, \ldots, n$) produces a single output (Y_i) based on a common production function employing m factors (X_i^k) ($k = 1, \ldots, m$) using the Cobb-Douglas production function with or without constant return-to-scale and Hicks-neutral technological progress.

$$\ln(Y_i) = b_0(Time) + \sum b_k \ln(X_i^k), \quad \sum b_k = V, \quad b_i \neq b_j. \tag{4-4}$$

The natural growth rate of kth factor (R_k) is exogenously given. The kth factor also moves between ith and jth member (X^k_{ij}) by an extended Gravity formulae:

$$X^k_{ij} = a_k(X^k_i X^k_j / X^k)\ln[((Y_j)^{(1/V)}/X^k_j)/((Y_i)^{(1/V)}/X^k_i)], \quad a_k < 1. \tag{4-5}$$

We define the total endowment of kth resource and the share of ith country as;

$$X^k = \sum X^k_i, \quad S^k_i = X^k_i/X^k. \tag{4-6}$$

Then $SY_i(0) = Y_i(0)/\sum Y_k(0) = [\Pi(S^k_i(0))^{b_k}]Y_a(0)/\sum Y_k(0)$, (4-7)

where $\ln(Y_a) = \sum b_k \ln(X^k)$. (4-8)

Then the growth rate of S^k_i is

$$R(S^k_i) = a_k X^k_j/X^k \ln[((Y_j)^{(1/V)}/X^k_j)/((Y_i)^{(1/V)}/X^k_i)] \text{ (sum by } j)$$
$$= a_k(L_k - \ln((Y_j)^{(1/V)}/X^k_j)), \tag{4-9}$$

where $L_k = \sum X^k_j/X^k \ln((Y_i)^{(1/V)}/X^k_i))$. (4-10)

From (4-4) to (4-9), $R(H_i) = \sum b_k L_k$ (sum by k), (4-11)

where $\ln(H_i) = \sum c_k \ln(S^k_i)$ (sum by k), $c_k = (b_k/a_k)$, $c_i \neq c_j$ $(i \neq j)$. (4-12)

Thus the share of $H_i(SH_i)$ is preserved over time,

$$R(SH_i) = 0, \; SH_i = H_i/H, \; H = \sum H_i. \tag{4-13}$$

When FPE holds in the long-run,

$$S_i(\infty) = S^k_i(\infty), \quad k = 1, \ldots, m, \quad i = 1, \ldots, n. \tag{4-14}$$

From (4-12) to (4-14),

$$SY_i(\infty) = H_i(0)^{(1/c)}/\sum(H_k(0))^{(1/c)}, \quad c = \sum c_k, \quad i = 1, \ldots, n. \tag{4-15}$$

Therefore the long-run output share is analytically calculable, and (4-7) and (4-15) define world mapping from $SY(0)$ to $SY(\infty)$ in an implicit way.

2. *Existence of eigenvector*

Equalizing (4-7) and (4-15), we have

$$[\Pi(S^k_i(0))^{b_k}]Y_a(0)/\sum Y_k(0) = H_i(0)^{(1/c)}/\sum(H_k(0))^{(1/c)}, \quad i = 1, \ldots, n. \tag{4-16}$$

Thus,

$$\Pi(S^k_1(0)/S^k_j(0))^{D_k} = 1, \quad j = 2, \ldots, n, \tag{4-17}$$
$$D_k = b_k[\sum b_m(1/a_m - 1/a_k)], \quad k = 1, \ldots, m, \quad \sum D_k = 0, \tag{4-18}$$
$$\sum S^k_i = 1 \text{ (sum by } i), \quad k = 1, \ldots, m. \tag{4-19}$$

Equations (4-17) and (4-19) define the eigenvector. As the degree of freedom

(DF) is ($nm - m - n + 1$) (> 0), basically an infinite number of non-trivial solutions exists. Also any trivial FPE satisfies (4-17) and (4-19).

(A) When two factors are used ($m = 2$), (4-17) deteriorates to

$$S_1^1(0)/S_j^1(0) = S_1^2(0)/S_j^2(0), \quad j = 2, \ldots, n, \tag{4-20}$$

and the only solution is a trivial FPE one with the inverse mapping F^{-1},

$$F^{-1}(SY(\infty)) = SY(0). \tag{4-21}$$

When $n = 2$, the dynamics of other solutions are:

$$\begin{aligned}&\text{if } a_1 >(<)\ a_2, \text{ so } D >(<)\ 0,\ D = (b_1 b_2/c)(1/a_2 - 1/a_1),\\ &\text{when } (X_1^1(0)/X_1^2(0))^D > (X_2^1(0)/X_2^2(0))^D,\\ &SY_1(0) >(<)\ SY_1(\infty),\ SY_2(0) <(>)\ SY_2(\infty).\end{aligned} \tag{4-22}$$

Therefore the long-run shares of world GDP change from the initial shares.

(B) When three factors are used ($m = 3$) with two members ($n = 2$), (4-17) reduces to

$$\begin{aligned}&Z_1^{D_1} Z_2^{D_2} Z_3^{D_3} = 1,\ Z_k = S_1^k(0)/S_2^k(0)S_1^k(0)+S_2^k(0) = 1,\\ &\quad k = 1, 2, 3, \quad 0 < Z_k < \infty, \quad D_1 + D_2 + D_3 = 0.\end{aligned} \tag{4-23}$$

The first equation of (4-23) implies a hyperplane convex to origin in three-dimensional space (Z_1, Z_2, Z_3). Let us check whether the mapping is many-to-one. When we set the ratio of $SY_1(0)/SY_2(0)$ as Q, the condition where the initial factor shares result in the same ratio (Q) is

$$Z_1^{b_1} Z_2^{b_2} Z_3^{b_3} = Q. \tag{4-24}$$

This also represents a hyperplane convex to origin in the same space. The trivial FPE, such as

$$Z_k = S_1^k(0)/S_2^k(0) = Q, \quad k = 1, 2, \tag{4-25}$$

satisfies (4-23) and (4-24). Because two hyperplanes have at least one common point, it implies that there is an infinite number of non-trivial FPE with the same initial output ratio (Q). So in this case, the mapping of (4-17) and (4-19) is many-to-one. As n increases, DF also increases, which gives us the next theorem:

THEOREM 2. *Existence and uniqueness of eigenvector: Under the current world economic system of (4-7) and (4-15), a unique implicit mapping and trivial FPE solution exist. While a non-trivial eigenvector does not exist when using two-factors ($m = 2$), there are infinite eigenvectors when $m > 2$ exists which constitutes many-to-one mapping.*

V. EMPIRICAL ESTIMATION OF A WORLD EQUATION

Now I will apply the PVU-ecomomy model to the current world. Tables I and II summarize the basic data of 1990 and 1996 respectively for the thirty-two largest countries in terms of GDP in 1990 and 1996 based on IMF sources. Each country

TABLE I

BASIC COUNTRY DATA, 1990

No. (1)	Name (2)	Rate (3)	GDPN (4)	GDP$ (5)	POP (6)	y$ (7)
1	U.S.A.	1.0	5,803	5,803	249.9	23,221
2	Japan	134.4	430,040	2,970	123.4	24,053
3	Germany	1.6	2,431	1,500	79.4	18,910
4	France	5.4	6,621	1,214	56.7	21,415
5	Italy	1,198.0	1,320,830	1,102	57.7	19,119
6	Canada	1.17	669	572	27.7	20,658
7	Australia	0.78	394	504	17.1	29,583
8	Spain	101.9	50,145	491	38.8	12,662
9	Brazil	0.024	11.549	481	114.7	4,194
10	Mainland China	4.78	1,831.95	383	1,155.3	331
11	U.K.	1.78	554.49	311	57.5	5,411
12	India	17.5	5,355.3	306	834.7	366
13	Netherlands	1.82	516.27	283	14.95	18,974
14	Mexico	2.81	738.9	262	82.59	3,183
15	Korea	707.8	178,797	252	42.87	5,892
16	Sweden	5.92	1,359.88	229	8.56	26,835
17	Switzerland	1.39	317.3	228	6.71	34,019
18	Belgium	33.42	6,593	197	9.97	19,787
19	Austria	11.37	1,813.48	159	7.73	20,633
20	Turkey	2,608.6	392,580	150	56.47	2,664
21	Finland	3.82	515.43	134	4.99	27,039
22	Denmark	6.19	825.31	115	5.14	25,939
23	Norway	6.26	722.21	82	4.24	27,209
24	Greece	158.51	13,143.1	74	10.16	8,161
25	Hong Kong	7.79	582.55	69	5.7	13,119
26	Portugal	142.56	9,855.1	58	9.9	6,982
27	Poland	0.95	56.027	52	38.12	1,547
28	Israel	2.02	105.831	74	4.66	11,242
29	Singapore	1.81	66.464	36	3.02	12,159
30	Hungary	63.21	2,089.3	33	10.35	3,193
31	Luxemburg	33.42	359.2	10	0.38	28,270
32	Iceland	58.28	364.4	6	0.25	25,010

Source: International Monetary Fund, *International Financial Statistics Yearbook 2000* (Washington, D.C., 2000).
Note: Column (1) = Rank of GDP in U.S. dollar. (2) = Name of country. (3) = Average exchange rate per U.S. dollar in the year. (4) = Nominal GDP in local currency; (5) = GDP in U.S. dollar converted by exchange rate in column (3); (6) = Size of population (million); (7) = Per capita GDP in U.S. dollar.

TABLE II

BASIC COUNTRY DATA, 1996

No. (1)	Name (2)	Rate (3)	GDPN (4)	GDP$ (5)	POP (6)	y$ (7)
1	U.S.A.	1.0	7,813	7,813	265.2	29,452
2	Japan	108.7	500,310	4,599	125.7	36,571
3	Germany	1.5	3,584	2,389	81.9	29,178
4	France	5.1	7,953	1,553	58.3	26,614
5	Italy	1,542.9	1,896,020	1,228	57.3	21,415
6	Mainland China	8.3	6,833	822	1,232	667
7	Brazil	1.01	788	748	157.8	4,740
8	Australia	0.78	521	668	18.31	36,529
9	Canada	1.36	833	613	29.9	20,459
10	Spain	126.6	73,743	582	39.2	14,825
11	Korea	804.4	418,479	520	45.5	11,423
12	U.K.	1.56	754	483	58.8	8,226
13	Netherlands	1.69	661	391	15.5	25,216
14	India	35.4	13,619	384	939.4	409
15	Mexico	7.6	2,503	329	96.5	3,411
16	Switzerland	1.24	365	295	7.0	41,729
17	Belgium	30.9	8,304	268	10.1	26,399
18	Sweden	6.71	1,756	261	8.8	29,610
19	Austria	10.59	2,453	231	8.0	28,742
20	Denmark	5.8	1,060	182	5.2	34,774
21	Turkey	81,404	14,772,100	181	65.5	2,768
22	Norway	6.4	1,020	158	4.3	36,106
23	Hong Kong	7.7	1,191	154	6.3	24,435
24	Poland	2.7	385	142	38.62	3,696
25	New Zealand	0.7	94.9	137	3.7	3,703
26	Finland	4.6	585	127	5.1	24,929
27	Greece	247.0	29,935	121	10.4	11,877
28	Portugal	154.2	16,808	108	9.93	10,974
29	Israel	3.19	308	96	5.7	16,938
30	Singapore	1.4	129	92	3.6	25,347
31	Czechoslovakia	27.1	1,572	57	10.3	5,613
32	Hungary	152.6	6,823	44	10.1	4,386

Source: International Monetary Fund, *International Financial Statistics*, July 2000.
Note: See Table I.

accounts for at least one per cent of total world GDP. Although thirty-two is a small number of countries, they are distributed across the world: eight Asian economies (China, India, Japan, the Republic of Korea, Singapore, Hong Kong, Israel, and Turkey), two Pacific countries (Australia and New Zealand), four American countries (the United States, Canada, Brazil, and Mexico), three Eastern European countries (Poland, Hungary, and Czechoslovakia), and fifteen Western European countries (Germany, France, United Kingdom, Italy, Norway, Sweden, Finland, the Netherlands, Belgium, Denmark, Austria, Switzerland, Spain, Portugal, and Greece).

(a) Because of a different estimation method, Russia was not included in the list. The country list follows the international arrangement of countries in 1990; therefore Hong Kong is separated from Mainland China, and Czechoslovakia is not divided yet.

(b) Luxemburg and Iceland in Table I were replaced by New Zealand and Czechoslovakia in Table II. When the L-index is compared between 1990 and 1996, these changes create a relatively minor error.

(c) GDP figures are nominal values. The inflation rate between 1990 and 1996 was mild for most of countries, so I assumed a 2 per cent rate of inflation, and deflated the 1996 GDP when I compare it with the 1990 GDP.

1. *Skewed GDP distribution*

(a) I divided the thirty-two sample countries into three groups: (Group-1) sixteen economies with per capita GDP of more than U.S.$20,000: the United States, Japan, Germany, France, Australia, the Netherlands, Switzerland, Belgium, Sweden, Austria, Denmark, Norway, Hong Kong, New Zealand, Finland, Singapore; (Group-2) eight countries with per capita GDP between U.S.$7,000–U.S.$20,000: Italy, Canada, Spain, Korea, the United Kingdom, Greece, Portugal, and Israel; (Group-3) eight countries with per capita GDP of less than U.S.$5,000: China, Brazil, India, Mexico, Turkey, Poland, Czechoslovakia, and Hungary. The proportion of world population for the three groups are 18.27 per cent, 7.48 per cent, and 74.25 per cent, respectively.

(b) The per capita GDP of Group-3 countries is lower than the world average per capita GDP of U.S.$6,673 (1996). Therefore its large proportion of world population (74.25 per cent) implies a very skewed GDP distribution. The per capita GDP differences are fairly big, and the coefficient of variation reached 2.46 in 1996. Such big differences will cause big changes in country shares of world GDP in the early stage of forecasting simulation.

2. *Estimation of capital stock*

I estimated the value of capital stock (K) as follows. Considering the recent technological progress and development of tertiary sector, I assumed a small economy-of-scale by assuming the elasticity of labor ($B_e = 0.40$) and of capital ($B_k = 0.60$), an economy-of-scale parameter of 1 per cent ($V = 1.01$), and the rate of Hicks-neutral technological progress as 1 per cent. I used the population as the surrogate of labor.

(a) Capital stock for 1990 ($K90$) was estimated by

$$\ln(K90) = (\ln(Y90) - V \cdot B_e \ln(E(90))/(B_k \cdot V), \quad (5\text{-}1)$$

(b) Capital stock for 1996 ($K96$) was estimated by

$$\ln(K96) = (\ln(Y96)/H - V \cdot B_e \ln(E(96))/(B_k \cdot V), \quad (5\text{-}2)$$

where, $H = 1.19405229$ ($= 1.03^6$). The resulting capital stock estimate ($K96$) is also in 1990 prices.

3. *Estimation of factor movement elasticity (A_e, A_k)*

Utilizing these figures, the ratios between 1990 and 1996, and resulting growth rates are as follows:

	1990 (1)	1996 (2)	(1)/(2) (3)	Growth (4)
Sum of labor	3,139.78	3,435.34	1.09413	0.015106
Sum of capital	1,414,689	1,938,366	1.37017	0.053891
Sum of GDP	18,215.34	22,924.76	1.25854	0.039069

The 5.98% average growth rate of world nominal GDP is approximately decomposed into rate of inflation (2%), growth of labor (0.90% = 1.51% × 0.6), and growth of capital (2.15% = 5.38% × 0.4), and technological progress (1%).

The values of the L-indices for 1990 and 1996 are calculated as follows:

	1990 (1)	1996 (2)	(2)/(1) (3)
L-index of labor (*LE*)	2.29083	2.13499	0.9319
L-index of capital (*LK*)	0.89279	0.86899	0.9733
L-index of GDP (*LY*)	1.74893	1.64488	0.9405

I assumed that each factor (E, K) grows at the same growth rate (R_e, R_k) in each country calculated above. This means that the decreases in the L-indices of labor and capital were due to the intercountry movement of factors to seek better remuneration. I assumed alternative values of elasticity (A_e, A_k), and repeated the simulation of the PVU-economy model from 1990 until 1996, and compared the predicted values of two L-indices for 1996 with actual values, and calculated the predicted sum of squared errors (PE).

$$PE = (LE(96) - 2.13499)^2 + (LK(96) - 0.86899)^2. \qquad (5\text{-}3)$$

Using a grid method, I changed the values of elasticity by 0.00001 units, and searched the pair (A_e, A_k) which minimized the prediction error. The errors for five cases were shown in Table III.

Case 2 showed the least prediction error, and the estimated values of the L-indices were as follows. The prediction errors were negligible.

Index	Actual Value (1996)	Estimated Value (1996)
LE	2.13499	2.13695
LK	0.86899	0.86506
LY	1.64488	1.64448

TABLE III

PREDICTION ERRORS

Case	A_e	A_k	PE
1	0.00013	0.00027	0.00003
2	0.00012	0.00027	0.00001
3	0.00012	0.00028	0.00002
4	0.00012	0.00026	0.00004
5	0.00011	0.00027	0.00003

Source: Calculated by author.

Therefore I adopted $A_e = 0.00012$ and $A_k = 0.00027$ as the elasticities for simulations. These estimates may not be so robust because I estimated with a final test between 1990 and 1996, but they exhibit two features. (A) These estimated values of elasticity are smaller compared with some estimates based on internal factor movements. For example, Fukuchi (1996, p. 31) estimated that $A_e = 0.02217$ and $A_k = 0.00579$ using data for nine regions of Japan. The order of magnitude implies that the intercountry factor movement is very slow, and would need 10,000 years to eliminate the differences in remuneration rates. (B) The movement elasticity of capital (A_k) two times bigger than that for labor. This contrasts with Fukuchi's estimates for Japan's economy ($A_e > A_k$), and accords with the expectation that the mobility of labor (capital) is higher (lower) in interregional (international) movement. The fact ($A_e < A_k$) implies that a labor-rich country will benefited and increase its long-run share of world GDP.

4. *Calculating the country share of world GDP*

The standard version of the basic model adopts the following values as parameters: $B_o = 0$, $B_e = 0.60$, $B_k = 0.40$, $V = 1.01$, $A_e = 0.00012$, $A_k = 0.00027$, $R_e = 0.0151$, $R_k = 0.0538$, and $PP = 0.01$.

World PVU (two factors, thirty-two countries)-economy model

- Exogenous growth rate: $R_k = 0.0538$, $R_e = 0.0151$. (5-4)
- Cobb-Douglas production function:

$$\ln(Y_i) = (0.40 \cdot \ln(K_i) + 0.60 \cdot \ln(E_i))^{1.01} + 0.01 \cdot (T), \quad i = 1, \ldots, 32. \quad (5\text{-}5)$$

- Intercountry factor mobility function:

$$E_{ji} = 0.00012 \cdot (E_j E_i / E) \cdot \ln[(Y_i/Y_j) \cdot (E_j/E_i)^{(1/1.01)}], \quad i, j = 1, \ldots, 32, \quad (5\text{-}6)$$

$$K_{ji} = 0.00027 \cdot (K_j K_i / K) \cdot \ln[(Y_i/Y_j) \cdot (K_j/K_i)^{(1/1.01)}], \quad i, j = 1, \ldots, 32. \quad (5\text{-}7)$$

• Bourguignon's inequality measure:

$$LE = -\sum(SE_i)\ln[SAY_i/(SE_i)^{(1/1.01)}], \quad (5\text{-}8)$$
$$LK = -\sum(SK_i)\ln[SAY_i/(SK_i)^{(1/1.01)}], \quad (5\text{-}9)$$
$$LY = (0.6 \cdot LE + 0.4 \cdot LK). \quad (5\text{-}10)$$

• Other definitions:

$$K_i = (1 + R_k) \cdot K_i(t-1) + \sum K_{ji}, \quad i = 1, \ldots, 32, \quad (5\text{-}11)$$
$$E_i = (1 + R_e) \cdot E_i(t-1) + \sum E_{ji}, \quad i = 1, \ldots, 32, \quad (5\text{-}12)$$
$$\ln YA = (0.4 \cdot \ln K + 0.6 \cdot \ln E) \cdot 1.01 + 0.01 \cdot T, \quad (5\text{-}13)$$
$$E = \sum E_j,\ K = \sum K_j,\ y_i = Y_i/YA,\ x_i = Y_i/K_i,\ y_a = YA/E,\ SE_i = E_i/E,$$
$$SK_i = K_i/K,\ SAY_i = Y_i/YA,\ \sum(SY_i) = \text{or} < 1, \quad i = 1, \ldots, 32. \quad (5\text{-}14)$$

The most important supporting hypothesis is that the PVU-economy model above applies equally to all thirty-two countries, which include many advanced as well as developing countries. The intensive argument over convergence has shown clearly that there has been convergence among the advanced countries, but many developing countries are not necessarily converging toward this advanced group.[5] Therefore, at least in the short-term, movement elasticity can be different for different pairs of countries. This is another simplification cost of the PVU-economy model.

Table IV shows the long-run world GDP share of each country. Columns (2) and (4) show the country and its long-run share in order of magnitude. Column (3) shows the shares in 1996. Column (5) shows the per cent of share change for each country compared with its share in 1996.

The map (F) from $SY(96)$ (column 3, $SY(0)$) to SYE(long run) (column 4, $SY(\infty)$) is the world equation,

$$F(SY(0)) = SY(\infty). \quad (5\text{-}15)$$

$SY(0)$ or $SY(\infty)$ is a thirty-two-dimensional vector. Because the number of factors is two, the mapping (F) is one-to-one, and inverse mapping (F^{-1}) exists as discussed in the previous section,

$$F^{-1}(SY(\infty)) = SY(0). \quad (5\text{-}16)$$

Some observations are:

(1) Thirteen countries increased their shares of world GDP while nineteen countries lost shares.

(2) Countries that increased their shares were China, India, Korea, and Turkey in Asia; Brazil and Mexico in Latin America; Poland, Czechoslovakia, and Hungary in Eastern Europe; Greece, Portugal, Spain, and United Kingdom in Europe.

5 For example, Zind (1991) showed the existence of different convergence groups.

TABLE IV

COMPARISON OF COUNTRY SHARES

No. (1)	Name (2)	*SY* (1996) (3)	*SYE* (Long-Run) (4)	(4) – (3) (5)
1	U.S.A.	0.3026	0.2294	(–) 0.0732
2	Japan	0.1781	0.1231	(–) 0.0550
3	Germany	0.0926	0.0705	(–) 0.0221
4	France	0.0602	0.0476	(–) 0.0126
5	Italy	0.0476	0.0414	(–) 0.0062
6	Mainland China	0.0318	0.1224	(+) 0.0906
7	Brazil	0.0299	0.0489	(+) 0.0190
8	Australia	0.0259	0.0179	(–) 0.0080
9	Canada	0.0238	0.0210	(–) 0.0028
10	Spain	0.0226	0.0229	(+) 0.0003
11	Korea	0.0201	0.0229	(+) 0.0028
12	U.K.	0.0187	0.0245	(+) 0.0058
13	Netherlands	0.0152	0.0123	(–) 0.0029
14	India	0.0149	0.0706	(+) 0.0557
15	Mexico	0.0128	0.0244	(+) 0.0116
16	Switzerland	0.0114	0.0075	(–) 0.0039
17	Belgium	0.0104	0.0083	(–) 0.0021
18	Sweden	0.0101	0.0077	(–) 0.0024
19	Austria	0.0090	0.0069	(–) 0.0021
20	Denmark	0.0071	0.0050	(–) 0.0021
21	Turkey	0.0070	0.0147	(+) 0.0079
22	Norway	0.0061	0.0043	(–) 0.0018
23	Hong Kong	0.0060	0.0049	(–) 0.0011
24	Poland	0.0055	0.0102	(+) 0.0047
25	New Zealand	0.0053	0.0037	(–) 0.0016
26	Finland	0.0049	0.0040	(–) 0.0009
27	Greece	0.0048	0.0054	(+) 0.0006
28	Portugal	0.0042	0.0049	(+) 0.0007
29	Israel	0.0037	0.0036	(–) 0.0001
30	Singapore	0.0035	0.0029	(–) 0.0006
31	Czechoslovakia	0.0022	0.0035	(+) 0.0012
32	Hungary	0.0017	0.0030	(+) 0.0013

Source: Calculated by author. Each country's share for 1996 was calculated from the data.

(3) As a whole, the developing country group increased shares, while the advanced country group lost the shares.

(4) The most remarkable feature was the thrust of East and South Asia (Mainland China and India) by 14.63 per cent, although this was predictable based on the huge size of the population of the two countries, and the relatively big elasticity in the movement of capital.

(5) The total long-run share for six Asian economies (China, Japan, India, Korea, Hong Kong, and Singapore) came to 34.68 per cent of total world GDP.

(6) Korea and Spain reserved their ranking while India, Mexico, and the United Kingdom ascended and Australia and Canada descended.

5. *Trend of L-indices*

I calculated the simulation experiment for the thirty-two-country model up to 100 years after 1996. The future trend of the L-indices are shown in Table V.

The per capita GDP differences (*LE*) steadily decline, while the per capital profit differences (*LK*) showed a minor increase. But the differences of remuneration rates as a whole (measure by *LY*) steadily declined over time, which indicates that the world system will move toward a long-run equilibrium position. The per capita GDP differences measured by the variation coefficient (*VC*) also steadily declined.

TABLE V

VALUES OF L-INDICES

Year (*J*)	*LE* (*J*)	*LK* (*J*)	*LY* (*J*)	*VC* (*J*)
2	2.3391	1.0503	1.8236	1.5767
10	2.3360	1.0516	1.8223	1.5733
20	2.3323	1.0532	1.8207	1.5690
30	2.3285	1.0549	1.8191	1.5647
40	2.3248	1.0566	1.8175	1.5605
50	2.3211	1.0582	1.8160	1.5562

Source: Calculated by author.

6. *Transitory trend of per capita GDP*

Table VI shows the trends of per capita GDP for the thirty-two countries. They steadily increase over time. But such a steady trend does not necessarily appear in every case. Table VII shows the trend of per capita GDP for each country over 100 years. The per capita GDP of rich countries such as the United States, Japan, Germany, and Australia decreases at first and needs 20–60 years to recover. The intercountry resource movement is the results of market-induced free decisions, but sometimes it shows such a Turnpike-type movement in which the growth paths of countries quickly approach a balanced growth path. Such movement appears when the intercountry per capita GDP differences or the movement elasticity are sufficiently big.

I repeated the simulations by changing the parameter values (A_e, A_k). As indicated in Table VIII, when parameter values are smaller, the Turnpike property disappears, and the per capita GDP of rich countries such as the United States, Japan, Germany, and Australia show steadily increasing trends.

TABLE VI

TREND OF PER CAPITA GDP ($A_e = 0.00012$, $A_\kappa = 0.00027$)

(1990 price)

No. (1)	Name (2)	Base Year (3)	10 Years (4)	20 Years (5)	30 Years (6)	40 Years (7)	50 Years (8)
1	U.S.A.	32.21	40.39	51.93	66.78	85.87	110.41
2	Japan	39.79	49.87	64.09	82.38	105.88	136.08
3	Germany	31.53	39.54	50.85	65.38	84.07	108.10
4	France	28.64	35.92	46.19	59.41	76.41	98.27
5	Italy	22.99	28.84	37.11	47.75	61.44	79.06
6	Mainland China	0.71	0.90	1.16	1.51	1.96	2.54
7	Brazil	5.22	6.56	8.47	10.93	14.11	18.21
8	Australia	38.98	48.86	62.80	80.71	103.73	133.33
9	Canada	21.81	27.37	35.22	45.32	58.32	75.04
10	Spain	15.80	19.83	25.54	32.88	42.34	54.52
11	Korea	12.16	15.27	19.67	25.35	32.65	42.07
12	U.K.	8.75	10.99	14.17	18.27	23.56	30.37
13	Netherlands	26.76	33.57	43.18	55.54	71.44	91.88
14	India	0.43	0.54	0.71	0.92	1.19	1.55
15	Mexico	3.61	4.55	5.87	7.58	9.80	12.65
16	Switzerland	44.17	50.34	71.11	91.38	117.41	150.87
17	Belgium	27.91	35.01	45.02	57.91	74.48	95.79
18	Sweden	31.30	39.25	50.47	64.89	83.44	107.29
19	Austria	30.35	38.05	48.93	62.92	80.91	104.05
20	Denmark	36.63	45.92	59.02	75.86	97.52	125.35
21	Turkey	2.91	3.67	4.74	6.13	7.92	10.23
22	Norway	37.98	47.60	61.18	78.64	101.08	129.91
23	Hong Kong	25.69	32.23	41.46	53.33	68.60	89.84
24	Poland	3.88	4.88	6.31	8.15	10.52	13.59
25	New Zealand	38.95	48.82	62.75	80.65	103.65	133.22
26	Finland	26.16	32.82	42.21	54.30	69.84	89.84
27	Greece	12.46	15.65	20.16	25.97	33.46	43.11
28	Portugal	11.50	14.44	18.61	23.98	30.89	39.81
29	Israel	17.73	22.25	28.64	36.87	47.46	61.10
30	Singapore	26.51	33.26	42.78	55.02	70.77	91.02
31	Czechoslovakia	5.84	7.35	9.48	12.23	15.78	20.37
32	Hungary	4.55	5.73	7.40	9.55	12.23	15.91

Source: Calculated by author.

7. *Speed of convergence*

In the PVU-economy model, the speed that shares of world GDP converge is defined using the shares at three points ($0, T, \infty$).

$$DSY_i(T) = (1/T)\ln[\ln(SY_i(0)/SY_i(\infty))/\ln(SY_i(T)/SY_i(\infty))]. \qquad (5\text{-}17)$$

The formulae above refer to GDP share, but a similar definition can be made for each factor (labor, capital) for each country. Because of the Turnpike-type nature,

TABLE VII

TREND OF PER CAPITA GDP

($A_e = 0.00012$, $A_k = 0.00027$, $R_e = 0.073$, $R_k = 0.05$, $PP = 0$, $B_e = 0.6$, $B_k = 0.4$, and $V = 1.01$)

(1990 price)

No. (1)	Name (2)	Base Year (3)	20 Years (4)	40 Years (5)	60 Years (6)	80 Years (7)	100 Years (8)
1	U.S.A.	32.21	32.82	33.40	33.86	34.20	34.39
2	Japan	39.79	39.38	38.98	38.60	38.20	37.75
3	Germany	31.53	32.17	32.78	33.27	33.63	33.84
4	France	28.64	29.58	30.49	31.26	31.87	32.30
5	Italy	22.99	24.44	25.90	27.19	28.28	29.17
6	Mainland China	0.71	1.16	1.89	2.86	4.10	5.57
7	Brazil	5.22	6.71	8.55	10.52	12.55	14.57
8	Australia	38.98	38.59	38.20	37.83	37.44	37.01
9	Canada	21.81	23.33	24.86	26.23	27.40	28.37
10	Spain	15.80	17.62	19.56	21.37	23.01	24.44
11	Korea	12.16	14.03	16.09	18.08	19.94	21.62
12	U.K.	8.75	10.53	12.58	14.64	16.65	18.54
13	Netherlands	26.76	27.84	28.90	29.80	30.53	31.08
14	India	0.43	0.75	1.28	2.05	3.07	4.34
15	Mexico	3.61	4.86	6.47	8.27	10.19	12.17
16	Switzerland	44.17	42.95	41.79	40.79	39.87	38.99
17	Belgium	27.91	28.86	29.78	30.56	31.18	31.62
18	Sweden	31.30	31.87	32.41	32.84	33.15	33.32
19	Austria	30.35	31.02	31.67	32.19	32.59	32.83
20	Denmark	36.63	36.51	36.37	36.21	36.00	35.72
21	Turkey	2.91	4.02	5.49	7.17	9.02	10.95
22	Norway	37.98	37.66	37.33	37.02	36.68	36.29
23	Hong Kong	25.69	26.84	27.97	28.94	29.74	30.36
24	Poland	3.88	5.17	6.81	8.64	10.57	12.54
25	New Zealand	38.95	38.49	38.03	37.59	37.16	36.68
26	Finland	26.16	27.26	28.34	29.26	30.01	30.58
27	Greece	12.46	14.31	16.33	18.27	20.08	21.71
28	Portugal	11.50	13.34	15.37	17.35	19.21	20.90
29	Israel	17.73	19.43	21.21	22.84	24.29	25.53
30	Singapore	26.51	27.56	28.59	29.47	30.18	30.71
31	Czechoslovakia	5.84	7.38	9.25	11.21	13.21	15.16
32	Hungary	4.55	5.93	7.66	9.54	11.49	13.45

Source: Calculated by author.

the definition on a country basis is sometimes difficult. The average convergence speed of labor, capital, and GDP (*DE*, *DK*, *DY*) is defined as a simple arithmetic average. When the speed becomes negative, I regard it as zero. The average speed of convergence for the shares of three factors are as follows. The speed for the share of labor (*DE*) steadily increases, while the speeds for the other two (*DK*, *DY*) show relatively volatile tendencies.

Table IX shows the initial share for 1996 (*SY*(0)), the share after 100 years

TABLE VIII

CONVERGENCE SPEEDS

Year (*J*)	*DE* (*J*)	*DK* (*J*)	*DY* (*J*)
10	0.000267	–0.000010	0.000116
20	0.000283	–0.000009	0.000123
30	0.000288	–0.000008	0.000126
40	0.000292	–0.000006	0.000128
50	0.000294	–0.000005	0.000129

Source: Calculated by author.
Note: Figures are absolute values.

TABLE IX

SPEED OF CONVERGENCE FOR SHARES OF WORLD GDP

No. (1)	Name (2)	*SY* (0) (3)	*SY* (100) (4)	*SYE* (0) (5)	*DY* (100) (6)
1	U.S.A.	0.30670	0.30578	0.22943	0.01048
2	Japan	0.17959	0.17871	0.12309	0.01301
3	Germany	0.09270	0.09243	0.07045	0.01079
4	France	0.06000	0.05988	0.04764	0.00939
5	Italy	0.04735	0.04734	0.04136	0.00216
6	Mainland China	0.03156	0.03250	0.12238	0.02194
7	Brazil	0.02958	0.02995	0.04891	0.02492
8	Australia	0.02562	0.02549	0.01791	0.01370
9	Canada	0.02347	0.02347	0.02105	–0.00092
10	Spain	0.02227	0.02233	0.02294	0.10076
11	Korea	0.01987	0.01998	0.02292	0.03638
12	U.K.	0.01847	0.01861	0.02454	0.02823
13	Netherlands	0.01492	0.01489	0.01229	0.00877
14	India	0.01464	0.01514	0.07056	0.02159
15	Mexico	0.01253	0.01272	0.02437	0.02350
16	Switzerland	0.01121	0.01114	0.00746	0.01489
17	Belgium	0.01018	0.01016	0.00825	0.01000
18	Sweden	0.00993	0.00990	0.00767	0.01194
19	Austria	0.00878	0.00875	0.00687	0.01158
20	Denmark	0.00692	0.00688	0.00500	0.01383
21	Turkey	0.00686	0.00698	0.01468	0.02292
22	Norway	0.00597	0.00594	0.00426	0.01420
23	Hong Kong	0.00582	0.00581	0.00490	0.00836
24	Poland	0.00538	0.00546	0.01020	0.02337
25	New Zealand	0.00519	0.00516	0.00366	0.01445
26	Finland	0.00481	0.00480	0.00403	0.00902
27	Greece	0.00468	0.00471	0.00539	0.03453
28	Portugal	0.00410	0.00412	0.00489	0.03148
29	Israel	0.00363	0.00363	0.00359	–0.17025
30	Singapore	0.00344	0.00343	0.00287	0.00962
31	Czechoslovakia	0.00217	0.00219	0.00346	0.02413
32	Hungary	0.00167	0.00169	0.00297	0.02330

Source: Calculated by author.
Note: The share speed is in per cent.

(*SY*(100)), the long-run equilibrium share (*SYE*) of country GDP, and the average income share convergence speed over 100 years (*DY* (100)).

(1) The share convergence speed differs for each country, and is distributed between zero and 0.1 per cent.
(2) The simple average of income share convergence speed is 0.01350 per cent.
(3) As shown in Table VIII, the convergence speed of labor productivity (*DE*) is about twice that of income convergence speed (*DY*), meaning that the income convergence speed is an average of factor convergence speed.

8. *Relative labor productivity convergence speed*

Sala-i-Martin (1994) pointed out that a 2 per cent convergence speed is typical for domestic factor movement and resulting per capita income convergence. When labor productivity diverges, the convergence speed of per capita GDP cannot be theoretically defined, so the empirical estimates were calculated by comparing productivity at two points of time (0 and *T*) without referring to the long-run value. In the current PVU-economy setting, labor productivity also diverges because I assumed $R_e < R_k$, and $PP > 0$, therefore a straight definition is impossible. But we can define the relative labor productivity (RLP) of *i*th country for *T*th year as the ratio between the *j*th country share of GDP and labor.

$$RLP(t, i) = SY(t, i)/SE(t, i). \qquad (5\text{-}18)$$

This variable will converge to unity in the long run. Thus the relative labor productivity convergence speed (DRLP) can be defined as follows. RLP in the long run is unity, so it can be neglected in the formulae. Table X shows the results of estimation.

$$DRLP_i(T) = (1/T)\ln[\ln(RLP_i(0))/\ln(RLP_i(T))]. \qquad (5\text{-}19)$$

(1) The distribution of estimated DRLP is as follows: eighteen countries recorded values between 0.0002–0.0003; six countries recorded DRLP greater than 0.0003; eight countries recorded DRLP less than 0.0002. So the mode exists between 0.0002 and 0.0003.
(2) Six countries recorded negative DRLP. This fact may be due to the Turnpike-type trend. But the absolute values are very small, so a definite conclusion whether they are anomaly will have to await further analysis.
(3) The simple average of DRLP is 0.02297 per cent. Compared with the usual 2 per cent rule for domestic interregional per capita GDP differences, the convergence speed of per capita GDP measured by DRLP indicates that the international convergence speed is only one hundredth as fast. The estimate is tentative, but it seems a 0.02 per cent rule prevails in the international economy in contrast to the 2 per cent rule prevailing within a country.

TABLE X
COMPARISON OF COUNTRY SHARES

No. (1)	Name (2)	*RLP* (0) (3)	*RLP* (100) (4)	*DRLP* (5)
1	U.S.A.	3.97178	3.83288	0.000261
2	Japan	4.90577	4.71347	0.000255
3	Germany	3.88852	3.75324	0.000264
4	France	3.53156	3.41506	0.000269
5	Italy	2.83503	2.75362	0.000284
6	Mainland China	0.08799	0.09169	–0.000171
7	Brazil	0.64366	0.64419	0.000019
8	Australia	4.80644	4.61815	0.000258
9	Canada	2.68972	2.06149	0.000289
10	Spain	1.94807	1.90629	0.000331
11	Korea	1.49926	1.47487	0.000413
12	U.K.	1.07893	1.06847	0.001372
13	Netherlands	3.30027	3.19490	0.000276
14	India	0.05354	0.05635	–0.000176
15	Mexico	0.44567	0.44929	–0.000101
16	Switzerland	5.44589	5.21845	0.000255
17	Belgium	3.44199	3.32900	0.000274
18	Sweden	3.85967	3.72429	0.000268
19	Austria	3.74197	3.61290	0.000270
20	Denmark	4.51660	4.34399	0.000262
21	Turkey	0.35959	0.36405	–0.000121
22	Norway	4.68285	4.50046	0.000261
23	Hong Kong	3.16840	3.06920	0.000280
24	Poland	0.47893	0.48203	–0.000088
25	New Zealand	4.80335	4.61376	0.000260
26	Finland	3.22635	3.12406	0.000279
27	Greece	1.53681	1.51062	0.000408
28	Portugal	1.41806	1.39613	0.000456
29	Israel	2.18598	2.13333	0.000317
30	Singapore	3.26955	3.16482	0.000279
31	Czechoslovakia	0.72079	0.71935	–0.000061
32	Hungary	0.56177	0.56346	–0.000052

Source: Calculated by author.

9. *Effects of changes in elasticity*

Because the elasticity values adopted are point estimates and may not be so robust, I repeated other simulations based on different sets of elasticity, and observed their effects on the long-run equilibrium shares. Table XI shows the results for the following three cases.

Case-1: The standard set (higher capital movement)

$A_e = 0.00012$, $A_k = 0.00027$;

Case-2: Equal elasticity of labor and capital

$A_e = 0.00012$, $A_k = 0.000135$;

TABLE XI

COMPARISON OF LONG-RUN COUNTRY SHARES OF WORLD GDP

No. (1)	Name (2)	*SY* (1996) (3)	*SYE* (Case 1) (4)	*SYE* (Case 2) (5)	*SYE* (Case 3) (6)
1	U.S.A.	0.3026	0.2294	0.2945	0.3341
2	Japan	0.1781	0.1231	0.1708	0.2192
3	Germany	0.0926	0.0705	0.0901	0.1017
4	France	0.0602	0.0476	0.0590	0.0632
5	Italy	0.0476	0.0414	0.0474	0.0448
6	Mainland China	0.0318	0.1224	0.0404	0.0299
7	Brazil	0.0299	0.0489	0.0330	0.0053
8	Australia	0.0259	0.0179	0.0248	0.0319
9	Canada	0.0228	0.0210	0.0237	0.0219
10	Spain	0.0226	0.0229	0.0230	0.0177
11	Korea	0.0201	0.0229	0.0209	0.0139
12	U.K.	0.0187	0.0245	0.0199	0.0109
13	Netherlands	0.0152	0.0123	0.0149	0.0155
14	India	0.0149	0.0706	0.0195	0.0019
15	Mexico	0.0128	0.0244	0.0144	0.0048
16	Switzerland	0.0114	0.0075	0.0109	0.0150
17	Belgium	0.0104	0.0083	0.0102	0.0109
18	Sweden	0.0101	0.0077	0.0099	0.0112
19	Austria	0.0090	0.0069	0.0087	0.0098
20	Denmark	0.0071	0.0050	0.0068	0.0085
21	Turkey	0.0070	0.0147	0.0081	0.0024
22	Norway	0.0061	0.0043	0.0059	0.0075
23	Hong Kong	0.0060	0.0049	0.0059	0.0060
24	Poland	0.0055	0.0102	0.0062	0.0022
25	New Zealand	0.0053	0.0037	0.0051	0.0066
26	Finland	0.0049	0.0040	0.0049	0.0050
27	Greece	0.0048	0.0054	0.0050	0.0034
28	Portugal	0.0042	0.0049	0.0044	0.0028
29	Israel	0.0037	0.0036	0.0038	0.0031
30	Singapore	0.0035	0.0029	0.0035	0.0036
31	Czechoslovakia	0.0022	0.0035	0.0025	0.0011
32	Hungary	0.0017	0.0030	0.0019	0.0007

Source: Calculated by author.

Case-3: Equal elasticity of labor and capital

$A_e = 0.00027$, $A_k = 0.00012$.

The comparison between Case-1 and Case-2 confirmed that when the elasticity of labor (capital) is higher than that for capital (labor), the relatively capital (labor) rich countries which has relatively higher (lower) capital intensity of labor or higher (lower) per capita GDP than average will increase their long-run shares of world GDP. When the two elasticities are equal, the long-run shares are roughly equal to the actual current shares although the lower income countries gain slightly.

VI. SUMMARY AND CONCLUSION

This paper undertook a modeling of the world economy based on a PVU(2, 32)-economy model. The tentative estimates for the elasticity of labor and capital using data for thirty-two countries for 1990 and 1996 showed that their order of magnitude is 1 to 160 compared with the estimates for intra-country elasticity. The estimate for the convergence speed of relative labor productivity based on a 100-year simulation implies that a 0.02 per cent rule prevails in the international economy, while the two per cent rule prevails within each country. But the long-run regional incidence of resource movement based on these estimates is quite big. With these tentative estimates, the long-run equilibrium shares of world GDP showed a strong China-India thrust.

Based on these simulations, PVU(2, 32)-economy model was shown to be a useful scheme for discussing the regional incidence of intercountry factor movement for long-run as well as the transitory shares of countries. Such a (1, *m*, *n*) scheme can be a useful tool for analyzing international factor movements, while the Ricardian (2, 2, 2) scheme is indispensable for studying international trade. The PVU-model also supplements interindustry table analysis from the supply side to further clarify the dynamic trend of the world economy.

There are various improvements that need to be made to the current PVU(2, *n*)-economy model. One shortcoming is that the stock of capital is defined as a mere aggregate, and neglects essential differences between hard or soft, and private or social capital. Another is that the endowment of capital stock is supposed to represent any intercountry technological differences.

Moving beyond the PVU-economy, the next challenge is to eliminate the "one good" assumption, and construct an (l, *m*, *n*) scheme where l, *m*, and *n* are positive integers. Fukuchi (2000b) pointed to a possible direction by employing a Chenery-type saturation equation, and calculating the long-run shares of world GDP for a (3, *m*, *n*) economy. The explicit introduction of intercountry distances is another future task.

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